

Strategic fiscal and monetary interactions in the Brazilian economy

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This paper identifies the leadership structure of the game played by monetary and fiscal authorities in the post-inflation targeting Brazilian economy. A small-scale New Keynesian model augmented with fiscal policy is estimated using Bayesian methods. I assume that monetary and fiscal authorities can act strategically under discretion in a non-cooperative setup and compare three different games: (i) simultaneous move; (ii) fiscal leadership; and (iii) monetary leadership. I find strong empirical support for the hypothesis that the fiscal authority acts as a Stackelberg leader. The results can shed some light to the improvement of policy design in the Brazilian economy.

Keywords. Monetary and fiscal policies, Strategic interactions, Policy games, Joint stabilization policies.

JEL classification. E52, E61, E62, E63.

1. Introduction

The Brazilian economy, in the aftermath of the global financial crisis of 2008, presents a compelling case to study the interactions between fiscal and monetary policies. In a period of only three years, from 2014 to 2016, the country experienced a quick fiscal deterioration, reversing half of the decrease in the public debt obtained previously, positioning itself amongst the most indebted economies in the world (Orair and Gobetti, 2017). As stressed out by Sims (2013) and Bai et al. (2017), the fiscal environment plays a key role in determining the effectiveness of monetary policies. Both level and structure of maturity of debt influence inflation determination. Hence, this recent lack of fiscal discipline combined with a single-minded inflation targeting may have created inflationary pressures, forcing the CPI (consumer price index) to breach double digits by the end of 2015 (Leeper and Leith, 2016). Nevertheless, the majority of the literature typically abstracts from the behavior of the fiscal authority, implicitly assuming that the only concern of fiscal policy is debt stabilization.

The empirical literature on monetary-fiscal interactions suggests that fiscal policy does more than just allow automatic stabilizers to operate (Auerbach, 2002; Favero and

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Monacelli, 2005). Since the works of Sargent and Wallace (1981) and Leeper (1991), joint stabilization problems had received more attention from macroeconomists. Most of the theoretical literature developed since then studies these matters assuming that policymakers operate with simple rules or in an optimizing framework in a cooperative setup with well-defined social objectives¹.

The adoption of simple rules requires that authorities are able to pre-commit to these rules (Currie and Levine, 1993) and do not have any clear links to policy objectives. However, Svensson (2003) argues that what we observe as rules are the equilibrium outcome of an optimization problem solved by the authority in charge. The assumption of complete cooperation in an optimizing framework, in its turn, is seldom realistic. It is more likely that authorities act in a strategic manner, since they do not necessarily cooperate on all targets (Fragetta and Kirsanova, 2010). For example, fiscal policy can assign a higher weight on output stabilization than the monetary authority, or might be concerned with debt stabilization. Monetary policy can be delegated to an inflation conservative Central Bank who puts a heavier weight on inflation than does the fiscal authority and society.

When both authorities are allowed to interact strategically, each policymaker has its own policy objective function and chooses its instrument to minimize the losses. Since authorities can assign different weights to their objectives or pursue different targets, this non-cooperation can lead to a conflict between monetary and fiscal policymakers. The result of this fight very much depends on how policy is conducted (under commitment or discretion), the choice of policy instruments and, specially, the sequencing of the game played between the two authorities. For example, Dixit and Lambertini (2003) and Blake and Kirsanova (2011) show that delegation of monetary policy to an inflation conservative central bank can make the equilibrium outcome suboptimal when both authorities play a simultaneous move (Nash) game. Furthermore, Kirsanova et al. (2005) show that the solution of a game where the fiscal authority acts as a Stackelberg leader Pareto dominates the Nash game when there is an excessive weight on output stabilization in fiscal objectives and/or the fiscal authority has a myopic behavior (discounts the future too much).

Ergo, what could be considered as a good policy design for one country may lead to welfare losses in another if the structure of the game played by both authorities is different. To address questions of good policy design, and similar issues, it is necessary to know the way the authorities actually interact with each other.

Motivated by these considerations, the main aim of this essay is to study empirically the strategic interactions between monetary and fiscal policies and identify the leadership regime that prevails in the game played by the two authorities in the Brazilian economy after the adoption of inflation targeting in 1999. I am unaware of any previous empirical research that aims to identify the leadership structure of monetary and fiscal interactions for the Brazilian case².

¹In this case, fiscal and monetary policies are both driven by a unique authority. See, e.g., Schmitt-Grohe and Uribe (2004a;b; 2007).

²The empirical literature on monetary and fiscal interactions for the Brazilian economy mainly focuses on identifying the prevailing regime of dominance between policies, see e.g. Fialho and Portugal (2005);

In order to do so, I use a small stylized standard dynamic New Keynesian model with monopolistic competition and sticky prices in the goods market, extended to include fiscal policy and nominal government debt, as proposed by [Blake and Kirsanova \(2011\)](#).

Some crucial assumptions about the nature of interactions between monetary and fiscal authorities are made, specifically: (i) both policymakers behave in a non-cooperative strategic manner, with non-identical objectives; and (ii) policy for both authorities is conducted under discretion. Regarding the degree of pre-commitment of authorities, I assume that policymakers act under discretion since the empirical literature shows that commitment policies are strongly dominated by discretion, for both monetary and monetary-fiscal regimes, ([Bai and Kirsanova, 2015](#))³.

As in [Fragetta and Kirsanova \(2010\)](#), the model is estimated using Bayesian methods. Different assumptions about the sequencing of the game lead to different methods of solution and linear feedback rules, which allow us to identify the leadership regime that best describes the behavior of the Brazilian economy in the analyzed period. Three different models are estimated and compared based on the Bayes factor: (i) a simultaneous move (Nash) game; (ii) a game where the fiscal authority acts as a Stackelberg leader; and (iii) a monetary leadership game⁴.

The empirical findings suggest that there is strong evidence in favor of a fiscal leadership in the Brazilian economy after the implementation of the inflation targeting regime. This result is in line with the *prior* belief since fiscal policy is made on a much lower frequency than monetary decisions. Besides, under an inflation targeting regime, monetary policy is expected to be credible and clear which allows the fiscal authority to exploit the reaction function of the monetary policymaker. This means that the Brazilian monetary policymaker, by acting as a follower, can discipline the fiscal policy ([Libich and Stehlík, 2012](#)). Moreover, estimation of policy objectives show that there are no evidences of neither output nor inflation conservatism by part of monetary or fiscal authorities. The major concern of the monetary policy is, as expected, inflation stabilization, while for the fiscal authority is the smoothing of its policy instrument.

This essay is structured in the following way. Section 2 outlines the model economy. In Section 3 we first discuss the choice of policy and instruments, the microfounded welfare metrics and all policy scenarios of interest. Section 4 discusses the econometric methodology, tests the leadership hypotheses, presents the empirical results and, finally, the impulse responses analysis. Section 5 concludes.

[Moreira et al. \(2007\)](#); [Ornellas and Portugal \(2011\)](#); [Lima et al. \(2012\)](#). Most closely related to the present essay, in terms of topics, is the work of [Saulo et al. \(2013\)](#). They study strategic interactions between fiscal and monetary policies in a model calibrated to the Brazilian economy after the implementation of the Real Plan. Nonetheless, they do not attempt to estimate the model and identify the leadership structure of the game played by policymakers.

³See, e.g., [Le Roux and Kirsanova \(2013\)](#); [Chen et al. \(2014; 2017\)](#) for, respectively, the UK, Euro area and US. And [Palma and Portugal \(2011\)](#) for monetary policy in Brazil.

⁴Unlike [Fragetta and Kirsanova \(2010\)](#), we do not disregard the monetary leadership regime *a priori* as implausible.

2. The Model Economy

We consider a standard dynamic New Keynesian model with monopolistic competition and sticky prices in the goods market, similar to those presented by [Woodford \(2003\)](#) and [Galí \(2008\)](#), extended to include fiscal policy and nominal government debt as proposed by [Blake and Kirsanova \(2011\)](#). There are two policymakers, a fiscal authority (the government) and a monetary authority (the central bank).

2.1 Households

The economy is populated by a representative infinitely-lived household who seeks to maximize the expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \zeta \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad (1)$$

subject to a standard sequence of flow budget constraints. $\beta \in (0,1)$ is household's discount factor, σ the inverse of intertemporal elasticity of substitution in consumption, φ is the inverse labour supply elasticity and ζ a relative weight on consumption of public goods. The aggregate variables C_t, G_t and N_t are, respectively, private consumption, government spending and labour supplied.

Maximization of (1) is subject to a conventional period budget constraint of the form:

$$P_t C_t + Q_t A_t \leq A_{t-1} + (1 - \tau) W_t N_t + T, \quad (2)$$

where W_t is the nominal wage, A_t is the nominal portfolio of one-period bonds at a price Q_t , T is a constant lump-sum tax or subsidy and τ is an exogenous income tax rate. Following [Dixit and Stiglitz \(1977\)](#),

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}} di \right]^{\frac{\varepsilon_t}{\varepsilon_t-1}}$$

is a consumption index with an elasticity of substitution between goods that varies over time according to some stationary stochastic process $\{\varepsilon_t\}$ ⁵. Finally,

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_t} di \right]^{\frac{1}{1-\varepsilon_t}}$$

is an aggregate price index.

Log-linearization around the deterministic steady state with zero inflation of first-order conditions and the national income identity allow us to obtain a dynamic IS equation^{6,7}:

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma} (i_t - \mathbb{E}_t[\pi_{t+1}]) - \mathbb{E}_t[\Delta \tilde{g}_{t+1}], \quad (3)$$

⁵By adopting a stochastic elasticity of substitution we allow for variations in desired price markups, which makes possible to generate shocks to the markups of firms, as in [Beetsma and Jensen \(2004\)](#).

⁶Lowercase letters denote log deviations of a variable from its steady-state value, $x_t = \log X_t - \log X$.

⁷In all derivations we follow [Woodford \(2003\)](#) and [Galí \(2008\)](#) who study closely related models.

where the endogenous variables are aggregate output y_t , government spending $\tilde{g}_t \equiv (G/C)(g_t - y_t)$, nominal interest rate i_t and inflation rate π_t .

2.2 Firms and price-setting

There is a continuum of identical monopolistically competitive firms, each of which produces a differentiated good using a production function given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where A_t is an exogenous time-varying level of technology, common to all firms, and $1 - \alpha$ is the labour-share.

We assume an AR(1) process for $\{a_t\}$:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \quad (4)$$

where $\rho_a \in [0,1]$ and $\{\varepsilon_t^a\}$ is a zero mean white noise process with constant variance σ_a^2 .

Price-setting is based on [Calvo \(1983\)](#) contracts where, at each period, only a fraction $1 - \theta$ of firms may optimally reset its prices. Hence, a fraction θ of firms keep their prices unchanged. Aggregation across prices yields a New Keynesian Phillips curve⁸:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(y_t - y_t^e) - \lambda \sigma \tilde{g}_t + \eta_t^\pi, \quad (5)$$

where $\kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$, $\lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \Theta$ and $\Theta = \frac{1 - \alpha}{1 - \alpha + \alpha\epsilon}$. η_t^π is a cost-push shock, which reflects variations in desired price markups ([Beetsma and Jensen, 2004](#)) or any other disturbance to marginal costs. The variable y_t^e denotes output in the efficient allocation (in the absence of nominal rigidities and distortionary cost-push shocks) and is given by:

$$y_t^e = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} a_t.$$

For the cost-push shock, it is assumed that it follows an exogenous AR(1) process:

$$\eta_t^\pi = \rho_\eta \eta_{t-1}^\pi + \varepsilon_t^\pi, \quad (6)$$

where $\rho_\eta \in [0,1]$ and $\{\varepsilon_t^\pi\}$ is a white noise process with constant variance σ_π^2 .

2.3 The government solvency constraint

Following [Blake and Kirsanova \(2011\)](#) and [Fragetta and Kirsanova \(2010\)](#), the government issues one-period nominal debt B_t in order to pay the principle and interests on the existing debt and to fund discrepancies between spending and tax revenues. The log-linearized government solvency constraint, or evolution of debt, can be written as:

$$\tilde{b}_t = \chi i_t + \frac{1}{\beta} \left[\tilde{b}_{t-1} - \chi \pi_t + \frac{\bar{C}}{\bar{Y}} \tilde{g}_t + \left(1 - \frac{\bar{C}}{\bar{Y}} - \tau \right) y_t \right], \quad (7)$$

⁸Following [Fragetta and Kirsanova \(2010\)](#), we assume that, in the efficient equilibrium, there are no solvency problems. This yields that, under the assumption of ζ constant, government spending in the efficient allocation is zero, $\tilde{g}_t^e = 0$, see [Galí and Monacelli \(2005\)](#).

where $\tilde{b}_t = \chi B_t / P_{t-1}$ ⁹, B_t is nominal debt stock, χ is the steady state debt to GDP ratio, $\bar{C}/\bar{Y}$ is steady state consumption to GDP ratio. A model-consistent value of τ can be obtained, given the steady state debt to GDP and consumption to GDP ratios, by $\tau = \chi(1 - \beta) + (1 - \bar{C}/\bar{Y})$.

A private sector rational expectations equilibrium consists of plan $\{y_t, \pi_t, \tilde{b}_t\}$ satisfying the dynamic IS equation, the New Keynesian Phillips curve and the evolution of debt equation, given the policies $\{i_t, \tilde{g}_t\}$, the exogenous processes $\{\eta_t^\pi, a_t\}$, and initial conditions $\tilde{b}_0$.

3. Policy Making

3.1 Choice of Policy and Instruments

Following current convention, we assume that the monetary policymaker uses the short-term nominal interest rate, i_t , as its instrument of policy. This is, indeed, the case for the Brazilian inflation targeting regime that uses the Selic (Special System of Clearance and Custody) as the primary instrument of monetary policy. The choice of fiscal instrument is more arbitrary, since there is no well-established form of fiscal policy rule (Blake and Kirsanova, 2011). For the Brazilian case, Castro et al. (2011) argue that changes in government spending take place more often than variations in tax rate, given that a large part of taxes are not allowed to move during the fiscal year. This mainly motivates our choice of government spending, $\tilde{g}_t$, as the fiscal authority's control variable.

In what regards the way agents' expectations are dealt with in the optimization problems, we assume that both policymakers adopt discretionary policies. Under discretion, policymakers can, and are expected to, reoptimize in each period, thus it is a time-consistent and credible policy (Fragetta and Kirsanova, 2010). The assumption of an optimal discretionary policy seems to be in line with the empirical evidence about the Brazilian monetary authority's preferences. In a paper by Palma and Portugal (2011), the authors show that the behaviour of the Brazilian economy, after the implementation of an inflation targeting regime, is better described by an authority acting under discretion when compared to the commitment optimal plan¹⁰.

When considering fiscal policy, however, there are no obvious reasons to expect that it has been conducted optimally (Chen et al., 2015). As pointed out by Fragetta and Kirsanova (2010), fiscal authorities are likely to be able to precommit to rules. Hence, by restricting ourselves to discretionary policies we are ruling out such possibility, as well as the time-inconsistent Ramsey policy, where the policymaker is able to credibly commit to a policy plan, and cases where policies are formulated in terms of simple rules¹¹.

⁹This definition allows us to work with the same model even if $\chi = 0$, see Blake and Kirsanova (2011).

¹⁰Similar results were found to the US and Euro-area economies by Chen et al. (2017; 2014) respectively, they found that monetary policy is best described as optimal and time-consistent, i.e. discretionary, rather than operating under commitment.

¹¹Rules-based policies are time-inconsistent and requires that policymakers are able to commit to the coefficients of rules. Besides, they are also non-strategic policies Fragetta and Kirsanova (2010).

3.2 Social Welfare

Following Rotemberg and Woodford (1998), Woodford (2003) and Blake and Kirsanova (2011), we assume that both authorities set their instruments to maximize a quadratic approximation (a second-order Taylor expansion) to the expected aggregate utility function of the representative household given by (1). The adoption of a quadratic loss function is quite attractive since that, given a system of linear restrictions, under this linear-quadratic framework we obtain linear policy rules. We show in Appendix B that this approximation implies that a benevolent policymaker minimizes the discounted sum of all future losses:

$$\mathbb{W} = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t^s, \quad (8)$$

with an intra-period loss function, W_t^s , given by:

$$W_t^s = \pi_t^2 + \tilde{\Phi}_Y(y_t - y_t^e)^2 + \tilde{\Phi}_G \tilde{g}_t^2 + \mathcal{O}(\|\xi\|^3), \quad (9)$$

where the weights $\tilde{\Phi}$ are functions of the structural parameters of the model¹² and are rescaled in order to normalize the coefficient on inflation to one, $\mathcal{O}(\|\xi\|^3)$ collects terms of higher order and terms independent of policy.

The expression (9) contains a quadratic term in $\tilde{g}$, this is due to the fact that the representative household derives utility from the consumption of public goods (Blake and Kirsanova, 2011).

3.3 Non-cooperative policies under discretion

If both fiscal and monetary authorities are benevolent, they use the same intra-period loss function (9) as their objective function to minimize the welfare loss (8) subject to the system (3)-(7).

The micro-founded coefficients of the intra-period loss function derived in the previous subsection place very tight cross-equation restrictions on the model, this can make the estimation problematic, but also, are thought to be implausible (Chen et al., 2015). Therefore, following Fragetta and Kirsanova (2010); Chen et al. (2015), we allow the weights on the objective functions of both policymakers to be freely estimate. In doing so, we assume that both authorities are not benevolent and can act non-cooperatively. By this we mean that there can be distortions between their targets and the social optimal values, such as an inflation conservatism of the monetary authority, or they can pursuit additional policy objectives that are not present in the social optimal objectives, such as instrument smoothing or a debt stabilization target in the fiscal authority objectives.

We assume that the monetary authority objective function is of the form:

$$W_t^M = \pi_t^2 + \Phi_{MY}(y_t - y_t^e)^2 + \Phi_{MG} \tilde{g}_t^2 + \Phi_{\Delta I}(\tilde{i}_t - \tilde{i}_{t-1})^2, \quad (10)$$

¹²See Appendix B.

where the weights attached to the output gap and government spending can be different from the social optimal. The reason for this can be policy delegation to a conservative monetary authority, or simply because the Central Bank cannot compute the social optimal. Besides, there is an additional interest rate smoothing target. This is motivated by the results of [Woodford \(2003\)](#) who shows that, under discretion, it is possible to reduce the ‘stabilization bias’ when the authority chooses to smooth movements in its instrument.

For the fiscal authority, our preferred specification, following [Fragetta and Kirsanova \(2010\)](#), is given by:

$$W_t^F = \pi_t^2 + \Phi_{FY}(y_t - y_t^e)^2 + \Phi_{FG}\tilde{g}_t^2 + \Phi_{\Delta G}(\tilde{g}_t - \tilde{g}_{t-1})^2 + \Phi_{FB}\tilde{b}_t^2, \quad (11)$$

where, as in the monetary objective, weights on output gap and government spending can deviate from the social optimal. Since fiscal policy is relatively inflexible - current period spending decisions are often based on past period allocations - the fiscal authority also have an additional government spending smoothing target ([Fragetta and Kirsanova, 2010](#)). Finally, we assume that the fiscal authority pursues a target for the stabilization of the public sector debt-to-GDP ratio, $\tilde{b}_t$, which is in accordance to the fiscal regime in place in Brazil since 1999 ([Castro et al., 2011](#)).

3.4 Strategic interactions

We allow the monetary and fiscal authorities to play strategic games with each other¹³. Specifically, we assume that the optimal problem outlined in the previous subsection is solved for two policymakers under three different structures of strategic interactions: (i) when the fiscal authority acts as a Stackelberg leader (fiscal leadership) in the policy game and chooses the best point in the monetary policymaker’s reaction function, (ii) the other way around, where the monetary authority acts as the leader (monetary leadership), or (iii) a simultaneous moves regime (Nash game).

For both the simultaneous move and the leader-follower case, we can describe the evolution of the economy given by equations (3)-(7) by the following linear system:

$$A_0 z_t = A_1 z_{t-1} + A_2 \mathbb{E}_t z_{t+1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_4 \mathbb{E}_t x_{t+1} + \tilde{A}_4 \mathbb{E}_{t+1} \tilde{x}_{t+1} + A_5 v_t, \quad (12)$$

where z_t is a vector of endogenous variables, x_t and $\tilde{x}_t$ are vectors of policy instruments for each policymaker and v_t a vector of stochastic disturbances.

The quadratic loss functions (10) and (11) can be rewritten as:

$$\mathbb{W}_1 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (y_t' W y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t), \quad (13)$$

$$\mathbb{W}_2 = \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t (y_t' \tilde{W} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t). \quad (14)$$

¹³But not with their future selves.

Table 1. Description of the data series used in estimation

Variable	Description	Source
Y_t	Gross Domestic Product	IBGE
G_t	Final consumption - Government	IBGE
i_t	Nominal interest rate - Selic (% per quarter)	BCB
π_t	CPI inflation: IPCA (% per quarter)	IBGE

Acronyms: IBGE - Brazilian Institute of Geography and Statistics; BCB - Central Bank of Brazil.

In the Appendix A we outline the solution procedure for optimal discretionary policies in a linear-quadratic rational expectations framework like (12)-(14), for both simultaneous moves and leader-follower cases. It is important to note that different cases of strategic interaction require different solution procedures and lead to different solutions for the linear-quadratic optimization problem described. Since the linear policy reaction for each policymaker is different between the Nash and the Stackelberg games¹⁴, we can identify the structure of the game played by the monetary and fiscal authorities in the Brazilian economy by performing a model comparison based on the marginal data density for each model.

4. Estimation

Following An and Schorfheide (2007) and Fragetta and Kirsanova (2010), the model is estimated using a system-based Bayesian approach¹⁵. The Bayesian estimation allows the use of additional information in the estimation process in the form of prior distributions. This prior information can add curvature to a likelihood function that can be flat along the dimension of weakly identifiable parameters. Moreover, as stated in Castro et al. (2011), the estimation of DSGE models for the Brazilian economy by classical full information methods can be even more difficult due to short span of the data sample.

4.1 Data Description

In order to identify the structure of the game played by monetary and fiscal authorities in the Brazilian economy, we estimated the model described on the previous subsections using four Brazilian data series as observable variables spanning from 1999.Q3 to 2019.Q1: real GDP, nominal interest rate, inflation and government spending to GDP ratio (see Table 1).

We have chosen to use data for the period after the adoption of the inflation targeting regime in Brazil, which was formally adopted on June, 1999. All data are at quarterly frequencies, seasonally adjusted, detrended and demeaned prior to estimation. Following Stock and Watson (1999), data were detrended using a one-sided version of the

¹⁴As shown in Appendix A, under a Stackelberg game the linear feedback function of the follower depends on the instrument of the leader. This is not true for the simultaneous moves game, where the reaction functions for both policymakers depend on the same set of variables.

¹⁵For Bayesian analysis of DSGE models see, for instance, An and Schorfheide (2007), Herbst and Schorfheide (2016) and Fernández-Villaverde et al. (2016).

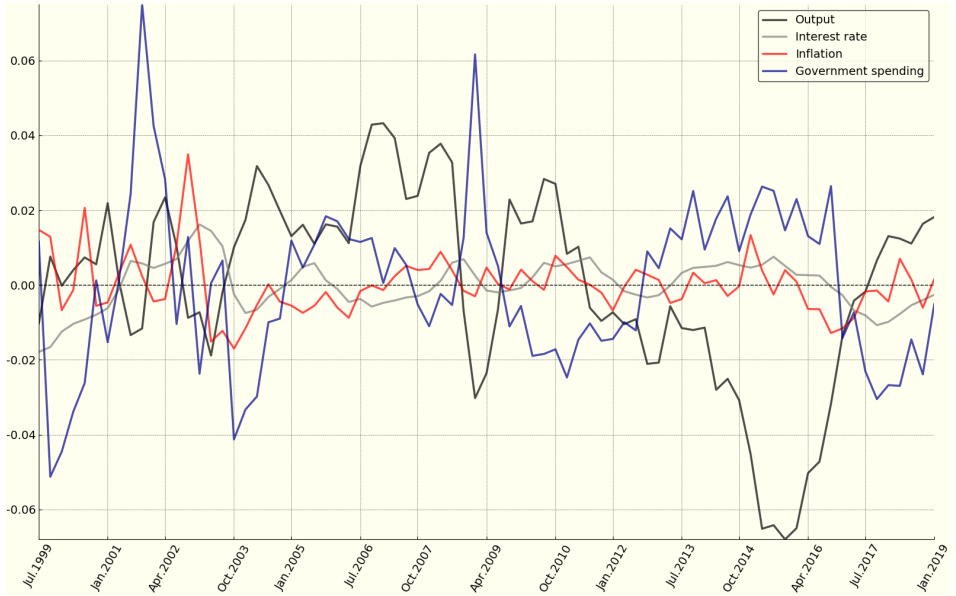


Figure 1. Time Series

Hodrick-Prescott 1997 filter¹⁶. Figure 1 depicts the resulting data series for the period analyzed.

4.2 Calibrated Parameters

To conduct our empirical analysis, we calibrate eight parameters. Table 2 presents the list of the calibrated parameters and their corresponding values to be used in the estimation of the model.

Based on the sample average calculated from the National Accounts, we set the government spending to GDP ratio in the steady state, $\bar{G}$, to 0.1919. The same procedure was used to obtain the debt to GDP ratio, χ , whose value matches the sample average of the total net public sector debt, obtained from the Brazilian Institute of Geography and Statistics (IBGE).

We calibrate three parameter values following the work of Castro et al. (2011) for the Brazilian economy: (i) the discount factor, β , is set to 0.989, which implies an annual steady state interest rate of approximately 4.4%; (ii) the mean of the stochastic elasticity of substitution, ε , is 11 which implies a 10% price markup; and (iii) the parameter α related to the labor-share in the production function is fixed in 0.448.

The autoregressive coefficient of the markup shock, ρ_η , is set to zero based on Fragetta and Kirsanova (2010).

Finally, from the set of calibrated parameters defined previously, we can calculate the model consistent values for the consumption to GDP ratio in the steady state, $\bar{C}$, and the constant income tax rate, τ .

¹⁶The smoothing parameter λ was set to 1600. To obtain the filtered series, we used data from 1996.Q1 to 2019.Q1 and discarded the observations from 1996.Q1 to 1999.Q2.

Table 2. Calibrated parameters

Parameter	Value	Description	Source
β	0.989	Discount factor	Castro et al. (2011)
$\bar{G}$	0.1919	Spending/GDP in steady-state	Sample average
$\bar{C}$	0.8081	Consumption/GDP in steady-state	Implied by the model
χ	0.3776	Steady-state debt-to-GDP ratio	Sample average
τ	0.1961	Constant income tax rate	Implied by the model
ρ_η	0.00	AR(1) coefficient of markup shock	Fragetta and Kirsanova (2010)
α	0.448	Related to the labor-share	Castro et al. (2011)
ε	11.00	Mean of markup shock	Castro et al. (2011)

4.3 Prior Distributions

The parameters on which we will perform the estimation can be divided in three major groups: (i) structural parameters; (ii) policy objectives coefficients - for both monetary and fiscal authorities; and (iii) shocks-related parameters - persistence and standard deviation of innovations. Table 3 presents the prior distribution for each parameter for both Stackelberg and simultaneous move cases. Whenever possible, we choose priors that are widely used in the literature on estimation of New Keynesian models and avoided using tight priors, since prior information is associated with a large degree of uncertainty.

Prior distributions of structural parameters are set following the work of Smets and Wouters (2007). Given its compact support, a Beta distribution with mean 0.5 and standard deviation of 0.1 is chosen for the Calvo parameter, θ . Normal distributions with, respectively, means of 1.50 and 2.00 are set for the two preference parameters estimated, the inverse of the intertemporal elasticity of substitution, σ , and the labour disutility parameter, φ .

In our preferred specification of policy objectives (10)-(11) there are two types of coefficients, the ones associated with the social optimal objective function and additional targets that each authority can pursuit. For the former, the coefficients related to output and government spending stabilization, we set the means of the prior distributions to match the implied theoretical values (given the priors for the structural parameters). For these coefficients we chose to use Beta distributions with loose priors that, given its compact support, allow us to exclude negative and unrealistically high values of weights. For the latter, we set a Beta distribution with mean 0.50 and standard deviation of 0.20 for the debt stabilization target for the fiscal authority and Gamma distributions with mean 0.70 for the smoothing instruments of both authorities¹⁷.

Finally, we follow Smets and Wouters (2007) to set the priors for the parameters related to the innovation processes. We assume a Beta distribution with mean 0.50 and standard deviation of 0.20 for the autoregressive coefficient of the technology process, and Inverse Gamma distributions with mean 0.10 and 2 degrees of freedom for the standard deviations of shocks.

¹⁷While Fragetta and Kirsanova (2010) assume Beta distributions for the smoothing coefficients, we chose Gamma distributions, given that those parameters have only a lower bound.

Table 3. Prior distribution

	Domain	Density	Mean	Std. Dev.
θ	$[0,1]$	Beta	0.50	0.10
σ	$\mathbb{R}_+$	Normal	1.50	0.37
φ	$\mathbb{R}_+$	Normal	2.00	0.50
Φ_{MY}	$[0,1]$	Beta	0.0317	0.020
Φ_{MG}	$[0,1]$	Beta	0.0292	0.015
$\Phi_{\Delta I}$	$\mathbb{R}_+$	Gamma	0.70	0.35
Φ_{FY}	$[0,1]$	Beta	0.0317	0.020
Φ_{FG}	$[0,1]$	Beta	0.0292	0.015
$\Phi_{\Delta G}$	$\mathbb{R}_+$	Gamma	0.70	0.35
Φ_{FB}	$[0,1]$	Beta	0.50	0.20
ρ_a	$[0,1]$	Beta	0.50	0.20
σ_a	$\mathbb{R}_+$	InvGamma	0.10	2
σ_η	$\mathbb{R}_+$	InvGamma	0.10	2
σ_r	$\mathbb{R}_+$	InvGamma	0.10	2
σ_g	$\mathbb{R}_+$	InvGamma	0.10	2

Table 4. Model comparison

Model	Log marginal data density	Bayes Factor
Fiscal Leadership	712.261	1.0
Simultaneous Move	708.926	0.0356
Monetary Leadership	707.807	0.0116

4.4 Model Comparison

Table 4 presents the posterior odds for cases of simultaneous moves, fiscal leadership or monetary leadership regimes. We treat each regime as equally probable by setting prior probabilities to one. Based on the Bayes factor, we found strong evidence¹⁸ in the estimation results that a fiscal leadership regime is dominant in the Brazilian economy for the period considered.

The likelihood that the data were generated under a simultaneous move regime is 0.0356 when compared to the case where the fiscal authority acts as a Stackelberg leader. For the monetary leadership regime, the Bayes factor is even smaller, which reflects the fact that monetary decisions take place in a much higher frequency than fiscal decisions (Fragetta and Kirsanova, 2010), which make it less likely for the Central Bank to exploit the reaction function of the fiscal policymaker.

4.5 Posterior Estimates

The posterior parameter estimates were computed by the Metropolis-Hastings sampling algorithm based on 100,000 draws¹⁹. According to the Geweke (1992) univariate diagnostic - see Appendix C - a sample of 100,000 was sufficient to ensure convergence of

¹⁸See Raftery (1995) for grades of evidence in Bayesian model selection.
¹⁹The acceptance rate was approximately 30% on average for each one of the estimated models.

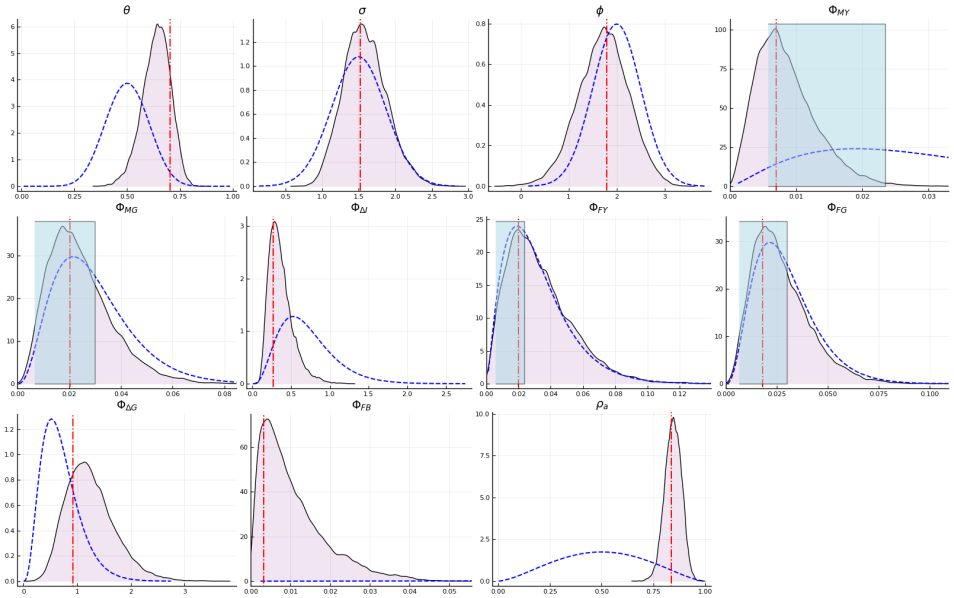


Figure 2. Prior and posterior distributions

the Metropolis-Hastings algorithm. Table 5 reports the posterior mean, standard deviation and the credible interval (5th and 95th percentiles) for the estimated parameters of each one of the models: simultaneous move (Nash), fiscal leadership (FL) and monetary leadership (ML). Figure 2 depicts the priors (dashed blue lines) and posteriors (shaded purple areas) of structural and policy objectives parameters for the dominant regime of fiscal leadership in the Brazilian economy²⁰.

Overall, there are no considerable differences between the estimated parameters in each one of the models. For large part of the parameters, observed data was informative in the estimation process²¹. Aside from the output stabilization target in the fiscal authority objective, the posterior distributions are more concentrated than the priors or are shifted to different points on the support (Figure 2).

Estimation of deep parameters of the models fall within plausible ranges. The estimated means of Calvo parameter (a measure of price stickiness), θ , implies that prices remain fixed, on average, for approximately three quarters indicating that price changes are as frequent as in most developed countries²². Moreover, its posterior distribution is tighter and shifted to the right when compared to the prior, which reflects the fact that observed data is informative along this dimension. The intertemporal elasticity of substitution in consumption, σ , obtained in the estimation procedure does not contradict

²⁰Dash-dotted red lines depict the estimated posterior mode, obtained by the direct maximization of the log of the posterior distribution with respect to the parameters.

²¹Çebi (2012) argues that, in this kind of DSGE models, it is a common finding that the means of prior and posterior distributions are similar.

²²Estimating an equally stylized model, Fragetta and Kirsanova (2010) found that prices are kept constant for between 3 quarters to one year for the US, UK and Sweden. Similarly, Smets and Wouters (2007) found an average duration of about 3 quarters of price contracts for the US economy.

Table 5.: Empirical posterior estimates

Parameter	Prior Distribution			Posterior Distribution											
	Mean	Std.		Simultaneous Move				Fiscal Leadership				Monetary Leadership			
				Mean	Std.	5%	95%	Mean	Std.	5%	95%	Mean	Std.	5%	95%
θ	0.5	0.1		0.6348	0.0722	0.5090	0.7464	0.6382	0.0662	0.5217	0.7400	0.6178	0.0737	0.4926	0.7353
σ	1.5	0.37		1.7087	0.3161	1.2102	2.2505	1.5878	0.2948	1.1238	2.0987	1.6650	0.3347	1.1518	2.2607
φ	2.0	0.5		1.6189	0.5613	0.6832	2.5217	1.7203	0.5284	0.8335	2.5730	1.6366	0.5543	0.7097	2.5384
Φ_{MY}	0.0317	0.02		0.0104	0.0058	0.0028	0.0213	0.0086	0.0046	0.0025	0.0171	0.0106	0.0062	0.0027	0.0226
Φ_{MG}	0.0292	0.015		0.0251	0.0129	0.0082	0.0493	0.0242	0.0123	0.0080	0.0476	0.0188	0.0096	0.0060	0.0371
Φ_{AI}	0.7	0.35		0.3638	0.1578	0.1589	0.6699	0.3641	0.1570	0.1670	0.6625	0.3839	0.1749	0.1682	0.6988
Φ_{FY}	0.0317	0.02		0.0311	0.0190	0.0075	0.0672	0.0332	0.0201	0.0075	0.0717	0.0315	0.0191	0.0074	0.0672
Φ_{FG}	0.0292	0.015		0.0285	0.0149	0.0093	0.0569	0.0270	0.0139	0.0088	0.0536	0.0279	0.0143	0.0092	0.0551
Φ_{AG}	0.7	0.35		1.1058	0.4248	0.5141	1.8824	1.2493	0.4525	0.6116	2.0779	1.0924	0.4203	0.5201	1.8768
Φ_{FB}	0.5	0.2		0.0173	0.0146	0.0015	0.0456	0.0106	0.0083	0.0015	0.0277	0.0111	0.0095	0.0009	0.0326
ρ_{μ}	0.5	0.2		0.8591	0.0384	0.7929	0.9202	0.8450	0.0400	0.7780	0.9079	0.8659	0.0397	0.7979	0.9288
σ_{μ}	0.1	2		0.0357	0.0067	0.0272	0.0477	0.0347	0.0058	0.0268	0.0455	0.0337	0.0057	0.0262	0.0445
σ_{η}	0.1	2		0.0194	0.0017	0.0169	0.0224	0.0196	0.0017	0.0170	0.0225	0.0195	0.0017	0.0169	0.0225
σ_{τ}	0.1	2		0.0168	0.0014	0.0148	0.0192	0.0168	0.0013	0.0147	0.0192	0.0167	0.0014	0.0147	0.0191
σ_{δ}	0.1	2		0.0172	0.0014	0.0151	0.0196	0.0172	0.0014	0.0151	0.0196	0.0171	0.0013	0.0150	0.0194

the findings of [Castro et al. \(2011\)](#) for the Brazilian economy, given that their results for this parameter fall within the 90% credible interval we obtain. Furthermore, estimates of φ are in line with the results of [Fragetta and Kirsanova \(2010\)](#) for the UK, US and Sweden, and imply an intermediate value of elasticity of labour supply.

Concerning the autoregressive process, there is a high degree of persistence on the technology shock, although the value of ρ_a is smaller than the one obtained by [Castro et al. \(2011\)](#).

Estimates of policy objective parameters suggest, in the period analyzed, that the preferences of both the Brazilian fiscal and monetary authorities are stable between the three regimes considered. As one would expect, the Central Bank of Brazil is more concerned with inflation stabilization than with other targets, once it puts a heavier weight on this objective. The other targets that the monetary authority pursuit in our specification are, in order of importance to the monetary authority, interest rate smoothing, government spending and output stabilization. The fiscal authority, in its turn, gives more attention to the smoothing of the fiscal policy instrument, followed by, respectively, inflation, output, government spending and debt stabilization. This reflects the fact that current decisions of fiscal policy are, indeed, based on past period allocations.

A closer inspection of Figure 2 makes evident that there is no evidence indicating neither inflation nor output conservatism by the monetary authority. In this figure, shaded rectangular areas show the theoretical distributions for the optimal social weights of policy parameters²³ that are based on the estimates obtained for the structural parameters of the model (with a 90% credible interval). Given that the posterior distributions of Φ_{MY} and Φ_{MG} overlap with the respective theoretical distributions, it is not possible to assert that those coefficients are different from the social optimal. Moreover, we do not find a substantial degree of interest rate smoothing by part of the Central Bank of Brazil²⁴. [Palma and Portugal \(2011\)](#) found that the weights on output stabilization, Φ_{MY} , and interest rate smoothing, $\Phi_{\Delta I}$, when the Central Bank of Brazil acts under discretion are, respectively, 0.01 and 0.2, which does not contradict our findings (see Table 5).

Just as for the monetary authority, estimates of fiscal policy preference parameters indicate that the values for Φ_{FY} and Φ_{FG} are not different from the social optimal weights. Although it is important to note that the marginal likelihood for output stabilization in the fiscal objectives, Φ_{FY} , is flat so that its posterior distribution agrees with the prior, while the posterior for Φ_{FG} is slightly more concentrated than the prior. Furthermore, we found a substantial weight on the government spending smoothing target, $\Phi_{\Delta G}$, being the major concern of the Brazilian fiscal authority. While the posterior of $\Phi_{\Delta G}$ is less concentrated than its respective prior, the mean of the distribution is shifted to the right on the support. Finally, there is little evidence of debt stabilization given that the estimated value of the posterior mean is small and the distribution is shifted to the lower end of the support, being very close to the origin.

²³Namely, parameters associated with output and government spending stabilization for both authorities.

²⁴[Fragetta and Kirsanova \(2010\)](#) found that the posterior means of this coefficient of 1.5, 0.8 and 0.9 for the UK, Sweden and US, respectively.

Summarizing our results, we found that the model which best describes the behavior of the Brazilian economy spanning from 1999.Q3 to 2019.Q1 is one of a fiscal leadership regime, where the fiscal authority acts as a Stackelber leader and chooses the best point on the reaction function of the central bank. There are no evidences indicating neither output nor inflation conservatism by part of monetary or fiscal authorities. As expected, the major concern of the monetary authority is inflation stabilization, while for the fiscal authority is the smoothing of the fiscal instrument. And the estimated weight of debt stabilization on the fiscal objectives is small, with the posterior distribution being shifted towards the origin.

4.6 Impulse Response Analysis

The dynamic properties of the model can be investigated by an impulse response analysis. Figure 3 displays the impulse response functions for the fiscal leadership regime²⁵ in terms of mean responses of the observable variables along with the unobservable debt accumulation and a 90% confidence interval.

Following a positive productivity shock, the efficient level of output increases and the efficient interest rate decreases. A higher efficient level of output reduces the marginal costs of the firms leading to a fall in inflation. At the time of the impact, monetary policy does not respond and the real interest rate initially rises. Afterwards, the reduction of the nominal interest rate causes the real interest rate to fall. This stimulates the economy, raising the output. Moreover, debt reduces following a higher output and a decrease on nominal interest rate. The fall of inflation and debt stock make the government responds with a small expansionary fiscal policy.

A positive markup shock raises inflation. In order to stabilize inflation, the monetary authority increases the nominal interest rate, which reduces output. The effect of a higher inflation on debt offsets the effect of a higher interest rate, which leads to an initial fall of the debt stock. Since the incentive to increase government spending in order to stabilize output practically offsets the opposite incentive to keep debt under control, fiscal policy does not change much (the fall of the fiscal instrument is very small).

An expansionary fiscal policy raises government spending and, consequently, output. Although an increase in government spending has a negative impact over inflation, the positive effect of a higher output prevails, increasing inflation. In order to keep inflation under control, the central bank raises the nominal interest rate. The increases in government spending and nominal interest rate raises the debt stock. Following this expansionary fiscal policy, the government subsequently reduces the government spending in order to stabilize debt. This contractionary policy is kept for long enough to bring output and inflation to the steady state level.

Finally, a positive shock to nominal interest rate lowers inflation and output. This higher interest rate leads to an increase on debt accumulation. In order to stabilize the debt stock, the fiscal authority reduces government spending. Hence, a tight monetary policy is followed by a tight fiscal policy.

²⁵Impulse responses for the other regimes are very much alike.

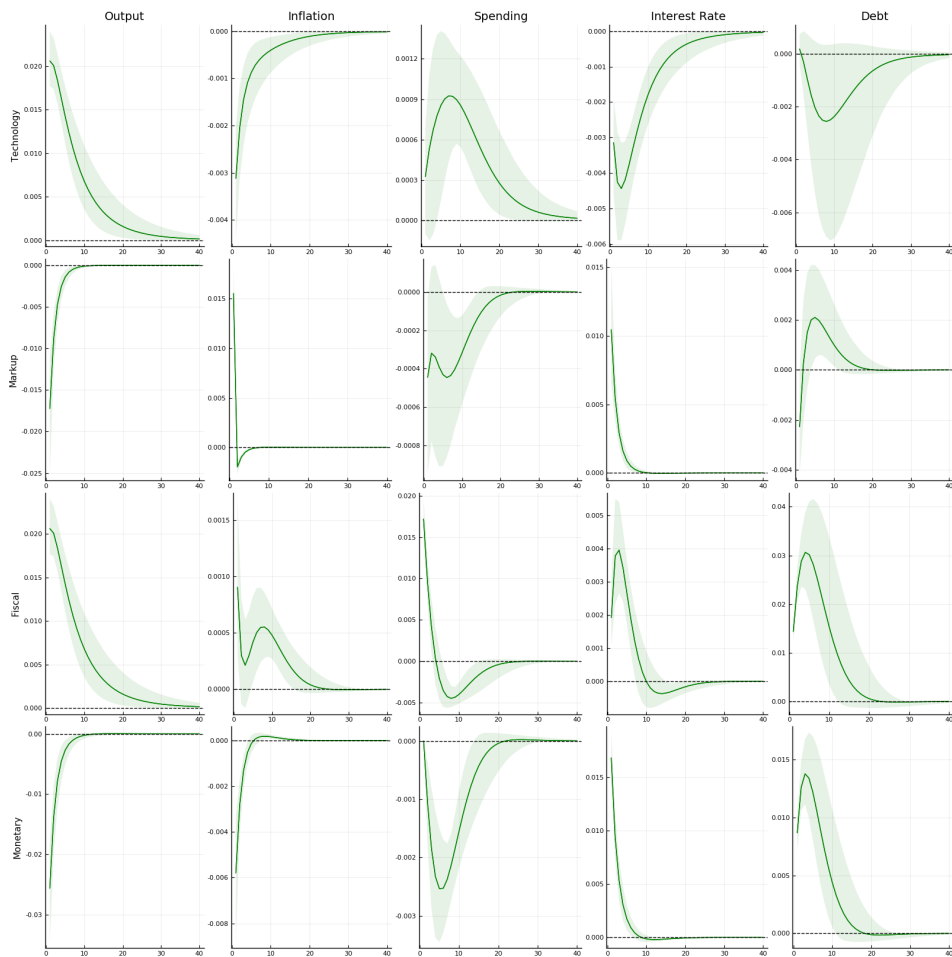


Figure 3. Impulse response analysis

5. Conclusion

This essay addresses empirically some questions of joint stabilization problems. A stylized small-scale New Keynesian model, extended to include fiscal policy and nominal government debt, is specified and estimated through Bayesian methods in order to identify the leadership structure of the game played by monetary and fiscal authorities in the Brazilian economy after the implementation of inflation targeting regime in 1999. Under the assumption that monetary and fiscal authorities can act non-cooperatively under discretion, Bayesian model comparison provides a strong empirical support for the hypothesis that the Brazilian monetary policymaker disciplines the fiscal authority. Put differently, a regime of fiscal leadership fits the data better than a simultaneous move game or the monetary leadership case.

Empirical estimates of policy objectives show that there are no evidence of inflation or output conservatism in the objective functions of both monetary and fiscal authorities. Moreover, as expected, we find that the greater concern of the Brazilian Central

Bank is inflation stabilization and that it assigns a very small weight to output stabilization. The fiscal authority, in its turn, operates with considerable smoothing of its instrument and shows little concern in stabilizing debt.

The analyses presented in this essay can shed some light to the improvement of policy design. The identification of the structure of the game played by the Brazilian monetary and fiscal authorities is important since it can help to mitigate the welfare losses caused by a potentially strategic interaction between them.

The behavior of the Brazilian economy over the last decades exhibit some events of possible conflicts between the monetary and fiscal authorities, as the hyperinflationary period studied by [Loyo \(1999\)](#) and the uprising inflation at the end of 2015. This suggests that the monetary policy not always works as a fiscal disciplinary tool. Thus, a fixed regime model, as considered in this essay, fails to capture these changes between conflict and cooperation in the conduct of policies. A possible extension to this work is to consider a model where optimal fiscal and monetary policies can switch over time, as done by [Chen et al. \(2015\)](#).

Further developments of this work include extensions of the model considered here to capture some characteristics of the Brazilian economy as the presence of administered prices, financially constrained households and an open economy.

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Appendix A: Model Solution Procedure

In this appendix we outline the solution procedure for optimal discretionary policies in a linear-quadratic (LQ) rational expectations framework. The methods employed here describe the solution for both the simultaneous move (Nash game) and the leader-follower (Stackelberg game) cases, and are taken from [Dennis and Ilbas \(2023\)](#). Unlike the framework developed by [Blake and Kirsanova \(2011\)](#), in [Dennis and Ilbas \(2023\)](#) the LQ optimal policy problem is put in a generalized structural form. By avoiding the matrix partitioning required by state-space methods, this procedure can be easily applied to larger models.

For both the simultaneous move and the leader-follower cases, the evolution of the economy can be described by the following linear system:

$$A_0 y_t = A_1 y_{t-1} + A_2 \mathbb{E}_t y_{t+1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_4 \mathbb{E}_t x_{t+1} + \tilde{A}_4 \mathbb{E}_t \tilde{x}_{t+1} + A_5 v_t, \quad (15)$$

where y_t is a vector of endogenous variables, x_t is the vector of policy instruments for one policymaker, $\tilde{x}_t$ is the vector of policy instruments of the other policymaker, $v_t \sim i.i.d.[0, \Omega]$ is a vector of stochastic disturbances, and matrices $A_0 - A_5$ contain the model's parameters²⁶.

The quadratic loss functions for the two policymakers are assumed to be given by:

$$\mathbb{W}_1 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (y_t' W y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t), \quad (16)$$

$$\mathbb{W}_2 = \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t (y_t' \tilde{W} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t), \quad (17)$$

where matrices W and Q contain the policy preferences of the policymakers and are symmetric and positive semi-definite. Both authorities are said to be in a cooperative setup if the following conditions are satisfied: $\beta = \tilde{\beta}$, $W = \tilde{W}$, $Q_1 = \tilde{Q}_1$ and $Q_2 = \tilde{Q}_2$. Equations (16) and (17) make clear that the objective function for each authority is allowed to depend on both policymaker's policy instruments, not just their own.

A.1 Simultaneous move

As discussed in [Dennis \(2007\)](#), quadratic objective functions like (16) and (17), with linear constraints, lead to linear decision rules. We assume that a stationary solution to the optimization problems exists and is given by:

$$y_t = H_1 y_{t-1} + H_2 v_t, \quad (18)$$

$$x_t = F_1 y_{t-1} + F_2 v_t, \quad (19)$$

$$\tilde{x}_t = \tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t. \quad (20)$$

Substituting this set of equations into the linear constraints, we can rewrite equation (15) as:

$$D y_t = A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t, \quad (21)$$

²⁶ A_0 is assumed to be a non-singular matrix.

where

$$D \equiv A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_t \tilde{F}_1. \quad (22)$$

Using the properties of convergent geometric series²⁷, we can rewrite the loss functions as follows²⁸:

$$\begin{aligned} \mathbb{W}_1 = & (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)' D'^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) \\ & + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t + \frac{\beta}{1-\beta} tr[(F_2' Q_1 F_2 + \tilde{F}_2' Q_2 \tilde{F}_2 + H_2 P H_2) \Omega], \end{aligned} \quad (23)$$

$$\begin{aligned} \mathbb{W}_2 = & (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t)' D'^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) \\ & + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t + \frac{\tilde{\beta}}{1-\tilde{\beta}} tr[(F_2' \tilde{Q}_1 F_2 + \tilde{F}_2' \tilde{Q}_2 \tilde{F}_2 + H_2 \tilde{P} H_2) \Omega], \end{aligned} \quad (24)$$

where

$$P = W + \beta(F_1' Q_1 F_1 + \tilde{F}_1' Q_2 \tilde{F}_1 + H_1' P H_1), \quad (25)$$

$$\tilde{P} = \tilde{W} + \tilde{\beta}(F_1' \tilde{Q}_1 F_1 + \tilde{F}_1' \tilde{Q}_2 \tilde{F}_1 + H_1' \tilde{P} H_1). \quad (26)$$

Differentiating the objective functions with respect to the vector of instrument variables for each policymaker gives us the following set of first order conditions:

$$\begin{aligned} \frac{\partial \mathbb{W}_1}{\partial x_t} &= A_3' D'^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + Q_1 x_t = 0, \\ \frac{\partial \mathbb{W}_2}{\partial \tilde{x}_t} &= \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + \tilde{Q}_2 \tilde{x}_t = 0. \end{aligned}$$

Since D , P and $\tilde{P}$ are implicit functions of the matrices in (18)-(20), the simultaneous move solution can be obtained as a fixed point in the following numerical procedure:

1. Initialize $H_1, H_2, F_1, F_2, \tilde{F}_1$ and $\tilde{F}_2$.
2. Compute D using equation (22), P using equation (25) and $\tilde{P}$ using equation (26).
3. Update $H_1, H_2, F_1, F_2, \tilde{F}_1$ and $\tilde{F}_2$ according to

$$\begin{aligned} F_1 &= -(Q_1 + A_3' D'^{-1} P D^{-1} A_3)^{-1} A_3' D'^{-1} P D^{-1} (A_1 + \tilde{A}_3 \tilde{F}_1), \\ F_2 &= -(Q_1 + A_3' D'^{-1} P D^{-1} A_3)^{-1} A_3' D'^{-1} P D^{-1} (A_5 + \tilde{A}_3 \tilde{F}_2), \\ \tilde{F}_1 &= -(\tilde{Q}_2 + \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} \tilde{A}_3)^{-1} \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_1 + A_3 F_1), \\ \tilde{F}_2 &= -(\tilde{Q}_2 + \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} \tilde{A}_3)^{-1} \tilde{A}_3' D'^{-1} \tilde{P} D^{-1} (A_5 + A_3 F_2), \\ H_1 &= D^{-1} (A_1 + A_3 F_1 + \tilde{A}_3 \tilde{F}_1), \\ H_2 &= D^{-1} (A_5 + A_3 F_2 + \tilde{A}_3 \tilde{F}_2). \end{aligned}$$

4. Iterate over steps 2-4 until convergence.

²⁷For further details, see the Appendix in Dennis (2007).

²⁸This transformation requires D to have full rank, which is satisfied since A_0 is a non-singular matrix.

A.2 Leader-follower

Without loss of generality, let us designate policymaker 1 as the leader and policymaker 2 as the follower. Besides, we assume that at each time t , the policymaker who acts as a follower observes the current decision rule x_t of the leader. Hence, given this assumption, the conjectured reaction function for the follower takes the form of a linear feedback function:

$$\tilde{x}_t = \tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t + Lx_t, \quad (27)$$

while the conjectured solutions for the private sector and the leader continue to be given by equations (18) and (19). The reaction function (27) implies that the follower takes into account the behavior of the leader when formulating its policy.

The solution procedure for the leader-follower case is similar to the one described for the simultaneous move case. Substituting the conjectured solutions into the linear constraints we obtain the same equation (21), but now D is given by:

$$D = A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_4 \tilde{F}_1 - \tilde{A}_4 L F_1. \quad (28)$$

The loss functions for the two policymakers are, then, given by:

$$\begin{aligned} \mathbb{W}_1 &= y_t' P y_t + x_t' Q_1 x_t + \tilde{x}_t' Q_2 \tilde{x}_t \\ &\quad + \frac{\beta}{1-\beta} \text{tr}[(F_2' Q_1 F_2 + (\tilde{F}_2 + L F_2)' Q_2 (\tilde{F}_2 + L F_2) + H_2 P H_2) \Omega], \\ \mathbb{W}_2 &= y_t' \tilde{P} y_t + x_t' \tilde{Q}_1 x_t + \tilde{x}_t' \tilde{Q}_2 \tilde{x}_t \\ &\quad + \frac{\tilde{\beta}}{1-\tilde{\beta}} \text{tr}[(F_2' \tilde{Q}_1 F_2 + (\tilde{F}_2 + L F_1)' \tilde{Q}_2 (\tilde{F}_2 + L F_1) + H_2 \tilde{P} H_2) \Omega], \end{aligned}$$

where

$$P = W + \beta(F_1' Q_1 F_1 + \tilde{F}_1' Q_2 \tilde{F}_1 + H_1' P H_1), \quad (29)$$

$$\tilde{P} = \tilde{W} + \beta(F_1' \tilde{Q}_1 F_1 + (\tilde{F}_1 + L F_1)' \tilde{Q}_2 (\tilde{F}_1 + L F_1) + H_1' \tilde{P} H_1). \quad (30)$$

After some algebraic manipulations and differentiating the two loss functions with respect to x_t and $\tilde{x}_t$, respectively, we obtain the following set of first order conditions:

$$\begin{aligned} \frac{\partial \mathbb{W}_1}{\partial x_t} &= (A_3 + \tilde{A}_3 L)' D^{-1} P D^{-1} [(A_1 + \tilde{A}_3 \tilde{F}_1) y_{t-1} + (A_3 + \tilde{A}_3 L) x_t + (A_5 + \tilde{A}_3 \tilde{F}_2) v_t] \\ &\quad + Q_1 x_t + L' Q_2 (\tilde{F}_1 y_{t-1} + \tilde{F}_2 v_t + L x_t) = 0, \end{aligned} \quad (31)$$

$$\frac{\partial \mathbb{W}_2}{\partial \tilde{x}_t} = \tilde{A}_3' D^{-1} \tilde{P} D^{-1} (A_1 y_{t-1} + A_3 x_t + \tilde{A}_3 \tilde{x}_t + A_5 v_t) + \tilde{Q}_2 \tilde{x}_t = 0. \quad (32)$$

The leader-follower solution can now be obtained as a fixed point of the following iterative procedure:

1. Initialize $H_1, H_2, F_1, F_2, \tilde{F}_1, \tilde{F}_2$ and L .

2. Compute D using equation (28), P using equation (29) and $\tilde{P}$ using equation (30).
3. Update $H_1, H_2, F_1, F_2, \tilde{F}_1, \tilde{F}_2$ and L according to

$$\begin{aligned}
F_1 &= -[Q_1 + L'Q_2L + (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_3 + \tilde{A}_3L)]^{-1} \\
&\quad \times (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_1 + \tilde{A}_3\tilde{F}_1 + L'Q_2\tilde{F}_1), \\
F_2 &= -[Q_1 + L'Q_2L + (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_3 + \tilde{A}_3L)]^{-1} \\
&\quad \times (A_3 + \tilde{A}_3L)'D'^{-1}PD^{-1}(A_5 + \tilde{A}_3\tilde{F}_2 + L'Q_2\tilde{F}_2), \\
\tilde{F}_1 &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}(A_1 + A_3F_1), \\
\tilde{F}_2 &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}(A_5 + A_3F_2), \\
H_1 &= D^{-1}(A_1 + A_3F_1 + \tilde{A}_3\tilde{F}_1 + \tilde{A}_3LF_1), \\
H_2 &= D^{-1}(A_5 + A_3F_2 + \tilde{A}_3\tilde{F}_2 + \tilde{A}_3LF_2), \\
L &= -(\tilde{Q}_2 + \tilde{A}_3'D'^{-1}\tilde{P}D^{-1}\tilde{A}_3)^{-1}\tilde{A}_3'D'^{-1}\tilde{P}D^{-1}A_3.
\end{aligned}$$

4. Iterate over steps 2-4 until convergence.

Appendix B: Social Welfare

We assume that the intra-temporal utility function is given by:

$$U(C_t, \zeta G_t, N_t),$$

and is separable in consumption, government spendings and hours (i.e., $U_{cn} = U_{cg} = U_{gn} = 0$).

A second-order Taylor expansion of U_t around the steady state allocation (C, G, N) yields:

$$\begin{aligned}
U_t - U &\simeq U_c C \left(\frac{C_t - C}{C} \right) + \zeta U_g G \left(\frac{G_t - G}{G} \right) + U_n N \left(\frac{N_t - N}{N} \right) \\
&\quad + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} \zeta U_{gg} G^2 \left(\frac{G_t - G}{G} \right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N} \right)^2.
\end{aligned}$$

In terms of log-deviations,

$$U_t - U \simeq U_c C \left(c_t + \frac{1 - \sigma}{2} c_t^2 \right) + \zeta U_g G \left(g_t + \frac{1 - \sigma}{2} g_t^2 \right) + U_n N \left(n_t + \frac{1 + \phi}{2} n_t^2 \right),$$

where $\sigma = -\frac{U_{cc}}{U_c} C = -\frac{U_{gg}}{U_g} G$ and $\phi = \frac{U_{nn}}{U_n} N$.

From the definition, we know that:

$$g_t = \frac{C}{G} \tilde{g}_t + y_t,$$

which implies

$$g_t^2 = \left(\frac{C}{G} \right)^2 \tilde{g}_t^2 + 2 \frac{C}{G} \tilde{g}_t y_t + y_t^2.$$

A second-order Taylor expansion of the national income identity yields:

$$c_t + \frac{1}{2}c_t^2 \simeq y_t + \frac{1}{2}y_t^2 - \frac{G}{C}(g_t - y_t) - \frac{1}{2}\frac{G}{C}(g_t^2 - y_t^2),$$

which can be rewritten in terms of $\tilde{g}_t$ as:

$$c_t + \frac{1}{2}c_t^2 \simeq y_t + \frac{1}{2}y_t^2 - \tilde{g}_t - \frac{1}{2}\frac{C}{G}\tilde{g}_t^2 - \tilde{g}_ty_t.$$

A first-order expansion of the national identity yields:

$$c_t = y_t - \tilde{g}_t,$$

then:

$$c_t^2 = y_t^2 - 2\tilde{g}_ty_t + \tilde{g}_t^2.$$

Hence, we have that:

$$c_t + \frac{1-\sigma}{2}c_t^2 \simeq y_t + \frac{1-\sigma}{2}y_t^2 - \tilde{g}_t - (1-\sigma)\tilde{g}_ty_t - \frac{1}{2}\left(\frac{C}{G} + \sigma\right)\tilde{g}_t^2. \quad (33)$$

For the government spending, we can write:

$$g_t + \frac{1-\sigma}{2}g_t^2 = \frac{C}{G}\tilde{g}_t + y_t + \left(\frac{1-\sigma}{2}\right)\left(\frac{C}{G}\right)^2\tilde{g}_t^2 + (1-\sigma)\frac{C}{G}\tilde{g}_ty_t + \frac{1-\sigma}{2}y_t^2. \quad (34)$$

To rewrite n_t in terms of output, we have:

$$(1-\alpha)n_t = y_t - a_t + \frac{\varepsilon}{2\Theta}\text{var}_i\{p_t(i)\},$$

which implies that²⁹:

$$n_t + \frac{1+\varphi}{2}n_t^2 = \frac{1}{1-\alpha}\left(y_t + \frac{\varepsilon}{2\Theta}\text{var}_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(y_t - a_t)^2\right). \quad (35)$$

Substituting equations (33)-(35) into the expansion of utility, we obtain:

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq y_t + \frac{1-\sigma}{2}y_t^2 - \tilde{g}_t - (1-\sigma)\tilde{g}_ty_t - \frac{1}{2}\left(\frac{C}{G} + \sigma\right)\tilde{g}_t^2 \\ &+ \frac{\zeta U_g G}{U_c C}\left(\frac{C}{G}\tilde{g}_t + y_t + \left(\frac{1-\sigma}{2}\right)\left(\frac{C}{G}\right)^2\tilde{g}_t^2 + (1-\sigma)\frac{C}{G}\tilde{g}_ty_t + \frac{1-\sigma}{2}y_t^2\right) \\ &+ \frac{UnN}{(1-\alpha)U_c C}\left(y_t + \frac{\varepsilon}{2\Theta}\text{var}_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(y_t - a_t)^2\right). \end{aligned}$$

Then, collecting terms:

$$\frac{U_t - U}{U_c C} \simeq \left[1 + \frac{\zeta U_g G}{U_c C} + \frac{UnN}{(1-\alpha)U_c C}\right]y_t + \left[\frac{\zeta U_g}{U_c} - 1\right]\tilde{g}_t$$

²⁹Excluding terms that are independent of policy.

$$\begin{aligned}
& + \frac{1}{2} \left[1 - \sigma + \frac{\zeta U_g G}{U_c C} (1 - \sigma) + \frac{UnN}{(1 - \alpha) U_c C} \frac{1 + \varphi}{(1 - \alpha)} \right] y_t^2 \\
& + \left[\frac{\zeta U_g}{U_c} (1 - \sigma) - (1 - \sigma) \right] \tilde{g}_t y_t + \frac{1}{2} \left[\frac{\zeta U_g C}{U_c G} (1 - \sigma) - \left(\frac{C}{G} + \sigma \right) \right] \tilde{g}_t^2 \\
& - 2 \frac{UnN}{(1 - \alpha) U_c C} \frac{1 + \varphi}{2(1 - \alpha)} y_t a_t + \frac{UnN}{(1 - \alpha) U_c C} \frac{\varepsilon}{2\Theta} \text{var}_i \{p_t(i)\}.
\end{aligned}$$

Now, if we assume that ζ is such that, in the steady state, $\zeta \frac{U_G}{U_C} = 1$, and $-\frac{U_n}{U_c} = MPN = (1 - \alpha) \frac{Y}{N}$, we can eliminate the linear terms and obtain:

$$\begin{aligned}
\frac{U_t - U}{U_c C} & \simeq -\frac{1}{2} \frac{Y}{C} \left[\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right] y_t^2 - \frac{1}{2} \frac{Y}{C} \left[\sigma \frac{C}{G} \right] \tilde{g}_t^2 \\
& + \frac{Y}{C} \frac{1 + \varphi}{1 - \alpha} y_t a_t - \frac{1}{2} \frac{Y}{C} \left(\frac{\varepsilon}{\Theta} \right) \text{var}_i \{p_t(i)\}.
\end{aligned}$$

Since the efficient level of output, in log-deviations from the steady state, is given by $y_t^e = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} a_t$, then, we can rewrite:

$$\frac{U_t - U}{U_c C} \simeq -\frac{1}{2} \frac{Y}{C} \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) (y_t - y_t^e)^2 + \sigma \frac{C}{G} \tilde{g}_t^2 \right].$$

Finally, making use of a result found in [Woodford \(2003\)](#), we can express the terms involving the price dispersion as a function of inflation:

$$\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_t(i)\} = \frac{\theta}{(1 - \beta\theta)(1 - \theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2.$$

Then, the welfare losses can be expressed as:

$$\mathbb{W} = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{Y}{C} \frac{\varepsilon}{\lambda} \pi_t^2 + \frac{Y}{C} \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) (y_t - y_t^e)^2 + \sigma \frac{Y}{G} \tilde{g}_t^2 \right],$$

where $\lambda = \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \Theta$.

Or, equivalently:

$$\mathbb{W} = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\lambda}{\varepsilon} \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) (y_t - y_t^e)^2 + \sigma \frac{\lambda}{\varepsilon} \frac{C}{G} \tilde{g}_t^2 \right]. \quad (36)$$

Appendix C: Convergence Diagnostic

This appendix presents the convergence diagnostics of [Geweke \(1992\)](#) for the estimated models. This diagnostic computes a normal-based test statistic comparing the sample means in two windows containing the initial 10% and the last 50% iterations. A non-significant p -value or, equivalently, a Z-score smaller than 1.96 in absolute value indicates convergence.

Table C.1 shows that there are evidences to rule out the possibility of non-convergence for all the three models estimated, given that for all the parameters we found non-significant p -values.

Table C.1. Geweke diagnostic

	Nash		FL		ML	
	Z-score	p-value	Z-score	p-value	Z-score	p-value
θ	−0.896	0.3702	1.830	0.0672	−1.354	0.1758
σ	0.516	0.6060	−0.028	0.9775	0.892	0.3722
φ	−0.607	0.5437	−0.596	0.5511	−0.952	0.3411
Φ_{MY}	1.157	0.2471	−0.958	0.3378	0.532	0.5945
Φ_{MG}	0.154	0.8776	1.112	0.2660	1.491	0.1358
$\Phi_{\Delta I}$	0.276	0.7829	0.038	0.9700	0.550	0.5822
Φ_{FY}	0.551	0.5814	−1.102	0.2704	−0.838	0.4019
Φ_{FG}	1.084	0.2785	−1.509	0.1313	−0.338	0.7356
$\Phi_{\Delta G}$	−0.447	0.6547	−0.396	0.6924	−0.008	0.9937
Φ_{FB}	−1.474	0.1404	−1.765	0.0776	1.894	0.0583
ρ_a	0.596	0.5513	−0.473	0.6359	1.248	0.2119
σ_a	−1.415	0.1570	0.976	0.3293	1.492	0.1356
σ_η	−0.476	0.6339	0.268	0.7887	−1.100	0.2714
σ_r	−1.807	0.0707	−0.424	0.6714	0.117	0.9066
σ_g	0.727	0.4674	−1.740	0.0819	1.359	0.1740