

# Optimal monetary and fiscal policy with limited asset markets participation and government debt

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## Abstract

Building on a standard New Keynesian model, the model economy is augmented to incorporate the government's budget constraint, where public expenditures are financed by distortionary taxation and/or issuing of long-term debt, and the existence of limited asset markets participation. Without the ability to commit to an optimal plan, discretionary policies in the presence of government debt yield the inflationary bias problem state-dependent and also creates a debt stabilization bias. Moreover, the presence of limited asset markets participation deepens the distortions in the economy. As a result of that, the size of the share of liquidity constrained agents impacts the long-run equilibrium values of relevant macroeconomic variables. Furthermore, the optimal response to shocks can be radically different for different values of government debt levels and fraction of rule-of-thumb consumers. Finally, higher levels of public debt causes a redistribution effect leading to rises in steady state inequalities amongst agents.

**Keywords:** Fiscal policy; Monetary policy; Limited asset markets participation; Time-consistency; Government debt.

JEL codes: E52, E61, E62, E63

## 1 Introduction

The onset of the global financial crisis in 2008 and its developments have reignited the debate concerning monetary and fiscal policy interactions that dates back, at least, to [Sargent and Wallace \(1981\)](#). Many countries reacted to the recent turmoil with joint policy actions that challenge the conventional macroeconomic policy prescription that insulates monetary and fiscal policies. To stimulate a weak macroeconomic environment, ambitious fiscal packages were launched causing a steadily increase in public debt-to-output ratios in advanced countries, as shown in [Figure 2](#).

The effects of the crisis resonated not only on public finances but also on the availability of credit to households. The empirical evidence on credit standards reported by [Albonico and Rossi \(2017\)](#) shows a sharp reduction of availability of credit to households in the economies of the Euro area and an even stronger decline for the US, see [Figure 3](#). This suggests that the fraction of the population who have no access to asset markets has increased during the period

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of the crisis. Nevertheless, limited asset markets participation seems to be an important feature of most economies even during normal times. A vast empirical literature has provided some evidence that the share of liquidity constrained households lies roughly between 30% and 50% of the agents population<sup>1</sup>.

This scenario of increasing government debt-to-GDP ratios and limited asset markets participation mainly motivates the current work, given that the presence of liquidity constrained consumers on the economy can have profound implications for macroeconomic policy prescriptions and that the level of debt, by its turn, may cause redistribution effects among those agents who have access to asset markets and those who are liquidity constrained.

Considering the aforementioned background, this paper studies jointly optimal and time-consistent monetary and fiscal policy in the presence of limited asset markets participation. In order to do so, we follow [Leeper et al. \(2019\)](#) in constructing a standard New Keynesian model extended to include optimally chosen distortionary taxation and government spending, when government debt can be issued with a more realistic maturity structure, but furthers their analysis by allowing that a fraction of households cannot smooth consumption intertemporally and, thus, consume all of their current disposable labor income at each period, in the tradition of [Galí et al. \(2004\)](#).

Our main results can be summarized as follows:

1. Increases in the share of liquidity constrained consumers are associated with substantial decreases in both the steady state rate of inflation and public debt-to-output ratio. Limited asset markets participation constitutes another source of distortion in the model economy. When the fraction of agents with no access to asset markets is higher, it raises per capita profits earned by optimizing households. This strengthens the state-dependent inflationary bias problem and the debt stabilization bias. Therefore, the discretionary policymaker has a stronger incentive to influence the endogenous inflationary bias and mitigate the costs of distortionary taxation through reductions in the level of government debt.
2. The presence of limited asset markets participation weakens the upward trend in steady state debt-to-GDP ratio caused by the issuing of longer-term government debt. Although a longer maturity structure of the debt pushes the public debt-to-output ratio up through reductions in the debt stabilization bias, when the fraction of liquidity constrained rises this effect is lower as this pushes the bias in the opposite direction. Therefore, the net effect is reduced and the upward trend in the steady state debt-to-output as maturity increases is flatter.
3. The optimal responses of policy instruments to a shock to the firms markup is largely dependent on both the share of liquidity constrained agents in the model economy and the size of government debt. Our numerical results show that for empirically plausible values of the fraction of rule-of-thumb consumers, when the public debt is low enough, the conventional policy prescription is reversed. In this case, the degree of activism of fiscal policy is enhanced and monetary policy accommodates the shock by ensuring that the real return on debt falls, yielding an inverse aggregate demand logic, as in [Bilbiie \(2008\)](#). However, when public debt breaches a given threshold, the conventional assignment of policies is restored.
4. Increases in the restriction on asset markets participation, when distortionary labor taxes are the main tool for fiscal stabilization, represent welfare gains for both optimizing and

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<sup>1</sup>See, e.g., [Campbell and Mankiw \(1989\)](#); [Muscatelli et al. \(2004\)](#); [Di Bartolomeo et al. \(2011\)](#); [Albonico et al. \(2014, 2016\)](#).

liquidity constrained agents. This is due to the fact that a higher share of liquidity constrained agents leads to a lower level of government debt in the long run equilibrium, which allows a smaller income tax rate. Rule-of-thumb agents, thus, experience a boost in consumption. For optimizing agents, additional to this effect, their financial wealth also rises. Therefore, although the welfare for both types of agents increases, steady state inequalities are also higher.

5. Finally, when there are shifts to a macroeconomic scenario of high level of government debt, for a given share of limited participation, there is a redistribution effect causing inequalities in the steady state to rise. The reason is the same presented in [Mankiw \(2000\)](#): higher levels of debt are associated with raises in labor income taxes. In our model economy, taxes fall equally among agents reducing their consumption. Nevertheless, optimizing agents experience a rise in their financial wealth that more than offsets the welfare losses caused by higher taxes. Thus, while liquidity constrained consumers suffer a welfare loss, there are gains for Ricardian agents, raising the long run equilibrium inequalities.

The paper is structured in the following way. Section 2 briefly reviews the related literature. The model economy is described on Section 3 and the optimal time-consistent policy problem in Section 4. The solution method and the model calibration are discussed in Section 5 and the numerical results are presented in 6. Section 7 concludes.

## 2 Related Literature

The current work is related to several branches of optimal monetary and fiscal policy and limited asset markets participation (LAMP henceforth) literatures. This section brings a brief overview of those works that are most closely related in terms of topics and methods.

There is a vast literature exploring the implications on monetary policy that the presence of limited asset markets participation, in the tradition of [Campbell and Mankiw \(1989\)](#), may cause. This strand of research tends to focus on the determinacy properties and policy design in standard New Keynesian models augmented by non-Ricardian agents. Extending a New Keynesian model with sticky prices to include rule-of-thumb consumers, [Galí et al. \(2004\)](#) found that the Taylor rule principle may not be a good policy guideline when some consumers are not able to smooth consumption intertemporally. If the central bank follows a Taylor rule responding to current inflation, the Taylor principle is strengthened when the share of rule-of-thumb consumers is high enough. Nevertheless, if the rule is set to respond to expected future inflation, rather than current, the policy rule may need to violate the Taylor principle, turning passive, in order to ensure a determinate equilibrium. [Bilbiie \(2008\)](#) by introducing limited asset markets participation to a standard dynamic general equilibrium model shows that this ‘inverted Taylor principle’ holds in general, regardless of whether the rule is specified in terms of current or expected inflation. In his model, when the share of liquidity constrained agents is high enough and/or the elasticity of labor supply is low, prevails an equilibrium where aggregate demand is positively related to real interest rates, i.e., the IS curve is upward sloping. Under this inverted aggregate demand logic, as he labeled, an optimal welfare-maximizing discretionary monetary policy has to follow a passive rule (lower real interest rate in response to a higher inflation) to guarantee uniqueness. In a similar vein, [Di Bartolomeo and Rossi \(2007\)](#) study the effectiveness of monetary policy in the presence of LAMP. Their main result is that, although an increase in the share of rule-of-thumb consumers reduces monetary policy effectiveness through consumption intertemporal allocation, the behavior of those same agents of reacting to changes in current disposable income in a Keynesian way supports a more effective

policy in a way that more than offsets the former negative effect. [Ascari et al. \(2017\)](#) show that the inverted aggregate demand logic of [Bilbiie \(2008\)](#) heavily relies on the assumption of nominal wage flexibility. Introducing wage stickiness to the model can restore the conventional policy prescription.

The intertwine between limited asset markets participation and fiscal policy mainly focus on attempts to overturn the inability of standard representative agents, based in the permanent income hypothesis, to replicate the empirical evidence that government expenditure shocks have positive effects on private consumption. Examples of that are, *inter alios*, [Galí et al. \(2007\)](#); [Furlanetto \(2011\)](#).

Most closely related to this work, there is also a literature on monetary and fiscal policy interaction allowing for the presence of non-Ricardian consumers. [Kirsanova et al. \(2007\)](#) study the importance of fiscal policy stabilization in a monetary union using an open-economy model with simple fiscal policy rules, rather than completely optimal, and non-Ricardian consumers based on the overlapping-generations framework of Blanchard-Yaari ([Yaari, 1965](#); [Blanchard, 1985](#)). Also considering the perpetual youth structure of Blanchard-Yaari in a New Keynesian model, [Chadha and Nolan \(2007\)](#) explore joint fiscal and monetary rules used in stabilization policy. Their main finding is that conducting a stabilization policy requires not only a monetary policy that adopts the Taylor principle, but also a fiscal policy that accounts for automatic stabilizers. Still building on the perpetual youth model of Blanchard-Yaari, [Leith and von Thadden \(2008\)](#) study determinacy properties of simple rules-based fiscal and monetary policy in the presence of non-Ricardian agents, which breaks the Ricardian equivalence. Analysis of local steady-state dynamics shows that stabilisation policies are dependent on the level of government debt. Unlike the present work, all studies mentioned use optimal exogenous policy rules. [Albonico and Rossi \(2017\)](#) investigate the effects that the presence of limited asset markets participation have on optimal monetary and fiscal policies when those are conducted by independent authorities who can act strategically. They assume that the government runs a balanced budget by levying on appropriate lump-sum taxes and that government debt plays no role. [Rigon and Zanetti \(2018\)](#) extend this framework to consider a microfounded welfare function to study optimal discretionary monetary policy and its interaction with fiscal policy in a New Keynesian model with non-Ricardian agents and government debt. In their linear-quadratic model they find that the welfare relevance of debt stabilization is proportional to the debt-to-GDP ratio, whose steady state value is exogenously given. When government debt is introduced to the model, it becomes a relevant state variable which needs to be accounted for, as [Leith and von Thadden \(2008\)](#) pointed out. Linearization around a steady state becomes problematic as now the steady state becomes endogenous and, thus, global techniques as applied in this work are more well suited.

Aside from the literature exploring the implications of limited asset markets participation, this work also connects to a more general literature on optimal fiscal and monetary policy. Under the scope of the latter, there is a vast number of works in the tradition of [Barro \(1979\)](#); [Lucas and Stokey \(1983\)](#) which tend to focus on real or flexible price economies where the policymaker operates under full-commitment (or Ramsey policies). [Barro \(1979\)](#) shows that, when the policymaker is not able to issue state-contingent debt or to use inflation surprises to hedge debt against shocks, debt and distortionary taxes are smoothed over time following a random walk process. In [Lucas and Stokey \(1983\)](#), by its turn, government can issue real state-contingent debt which can isolate government's finances from the effects of shocks and, thus, taxes are flat and inherit the properties of disturbances, unlike the tax-smoothing result of [Barro \(1979\)](#). In both the aforementioned works, the long-run level of debt depends on its initial conditions and, therefore, fail to explain why the public debt is a sizable share of output. [Aiyagari et al. \(2002\)](#) show that a departure from the assumption of complete markets in a

model otherwise identical to [Lucas and Stokey \(1983\)](#) makes this issue even worse, as it now government accumulate assets rather than liabilities.

The inability of those models to allow for debt accumulation can be overturned through departures from the assumption of benevolent planners under full-commitment. The political economy literature shows that when there is a political disagreement, which is a limitation to the ability to commit, among different policymakers, this can lead to an inefficiently high level of government debt ([Persson and Svensson, 1989](#); [Alesina and Tabellini, 1990](#)). This gave rise to a literature focusing on optimal time-consistent fiscal policy in real models. The assumption of lack of commitment by policymakers yields their optimal behavior characterized by generalized Euler equations that involves the derivatives of some equilibrium decision rules and, thus, linearization around a deterministic steady state becomes a complicate matter as now this steady state is endogenous and a priori unknown. This fact claims for the application of different numerical techniques in order to solve the model, see [Klein and Ríos-Rull \(2003\)](#); [Krusell et al. \(2006\)](#); [Ortigueira \(2006\)](#); [Klein et al. \(2008\)](#). [Debortoli and Nunes \(2013\)](#), using a numerical algorithm that is similar to the one used in the present work, find that the lack of commitment is enough to ensure that the government debt converges to a specific and determinate level of steady state (a striking different result from the full commitment case). Nevertheless, the economy often converges to a steady state characterized by no debt accumulation at all and, therefore, lack of commitment *per se* cannot help to explain why the level of debt is so high in several developed economies.

Finally, the work that is most closely related to ours, both in topics and numerical methods, was developed by [Leeper et al. \(2016, 2019\)](#). In their paper they study jointly optimal and time-consistent monetary and fiscal policy in a standard New Keynesian model with optimally chosen distortionary taxation and government spending, augmented to include government debt with a more realistic maturity structure. Their model is solved using non-linear projection methods, more specifically, using Chebyshev polynomials. Their main result is that the standard inflationary bias problem, under discretion, becomes state-dependent and it is exacerbated when the level of public debt is high and/or debt is short-term. But this also creates a debt stabilization bias as now the policymaker incentive to return government debt to its steady state value is higher. Those considerations can change radically the optimal response to shocks in New Keynesian models. In this work, we build on the models of [Leeper et al. \(2016, 2019\)](#) to incorporate another source of departure from Ricardian equivalence, alongside distortionary taxation. That is, we extended their work to allow for limited asset markets participation.

### 3 The Model

Building on a standard New Keynesian model, our model economy is augmented to incorporate the government’s budget constraint, where government spending is financed by distortionary taxation and/or issuing of long-term debt, and the existence of limited asset market participation.

#### 3.1 Households

The model economy is populated by a continuum of infinitely-lived households of unit mass. A fraction  $1 - \lambda$  is represented by optimizing households who are forward looking, can smooth consumption and have access to asset markets: we refer to this subset of consumers as ‘optimizing’ or ‘Ricardian’ agents<sup>2</sup> ([Galí et al., 2004, 2007](#)). The remaining  $\lambda$  share of households

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<sup>2</sup>Alternative nomenclatures found in the literature are: ‘asset holders’ in [Bilbiie \(2008\)](#), ‘savers’ in [Mankiw \(2000\)](#), ‘market participants’ or ‘active’ in [Nisticò \(2016\)](#).

is composed of agents who do not own any assets, cannot smooth consumption and, therefore, display a “hand-to-mouth” behavior of consuming all their current disposable labor income at each period: ‘liquidity constrained’ (Bilbiie, 2008) or ‘rule-of-thumb’ consumers<sup>3</sup> (Galí et al., 2004, 2007). As it is typically assumed in the LAMP literature, the fraction  $\lambda$  is an exogenously given constant, meaning that agents cannot change their type over time<sup>4</sup>.

All households derive utility from private consumption,  $C_t^j$ , the provision of public goods,  $G_t$ , and disutility from labor supply,  $N_t^j$ , where  $j \in \{o, r\}$ <sup>5</sup>. Both optimizing and liquidity constrained agents share the same preference structure defined by the discount factor  $\beta \in (0, 1)$  and period utilities that take the following separable form:

$$u(C_t^j, G_t, N_t^j) = \frac{(C_t^j)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^j)^{1+\varphi}}{1+\varphi}, \quad (1)$$

where  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution between private consumption,  $\sigma_g > 0$  is the inverse of the intertemporal elasticity of substitution between public consumption,  $\chi > 0$  is a scaling parameter, and  $\varphi > 0$  is the inverse of the Frisch labor supply elasticity.

### Ricardian households

Ricardian agents seek to maximize the expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^o)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \right), \quad (2)$$

subject to the sequence of budget constraints:

$$P_t C_t^o + P_t^M \frac{B_t^M}{1-\lambda} \leq \frac{\Xi_t}{1-\lambda} + (1 + \rho P_t^M) \frac{B_{t-1}^M}{1-\lambda} + (1 - \tau_t) W_t N_t^o, \quad (3)$$

where  $P_t$  is the nominal price index,  $\Xi_t$  is the Ricardian’s share of profits in the monopolistically competitive firms,  $W_t$  is the nominal wage set in a competitive labor market, and  $\tau_t$  is a wage income tax rate. Following Woodford (2001), optimizing households buy government bonds  $B_t^M$  in period  $t$  at price  $P_t^M$  that are actually a portfolio of many bonds paying an exponential decaying coupon of  $\rho^j$  monetary units  $j+1$  periods after they were issued<sup>6</sup>, where  $0 \leq \rho < \beta^{-1}$ . If prices are stable, a measure of the duration of such bond is given by  $(1 - \beta\rho)^{-1}$ . The budget constraint (3) states that the total financial wealth of Ricardian agents in period  $t$  plus its consumption spending cannot exceed the sum of financial wealth brought into the period and the after-tax nonfinancial income.

It is important to emphasize that policymakers cannot levy on any kind of lump-sum tax-financed subsidy that would enable him to offset distortions associated with monopolistic competition, a typical, but unrealistic, assumption in New Keynesian models (Leeper et al., 2016, 2019).

<sup>3</sup>Or ‘non-traders’ as in Alvarez et al. (2001), ‘spenders’ in Mankiw (2000) and ‘financially inactive’ in Nisticò (2016).

<sup>4</sup>A possible way to generalize this framework is to allow for stochastic transition between the two types of agents, see Nisticò (2016). We leave the introduction of this Markov switching process to future research.

<sup>5</sup>Henceforth, to keep the notation of Galí et al. (2004, 2007), superscripts “o” and “r” denote, respectively, variables of optimizing and rule-of-thumb households.

<sup>6</sup>Note that this simple structure nests the cases of consols ( $\rho = 1$ ) and the standard single period bonds ( $\rho = 0$ ).

Necessary and sufficient conditions for household optimization require the household's budget constraints to bind with equality. Defining Ricardian consumers' wealth as  $D_t \equiv (1 + \rho P_t^M) \frac{B_{t-1}^M}{1-\lambda}$ , the no-Ponzi game constraint can be written as:

$$\lim_{T \rightarrow \infty} \mathbb{E}_t \left[ R_{t,T}^M \frac{D_T}{P_T} \right] \geq 0, \quad (4)$$

where  $R_{t,T}^M \equiv \prod_{s=t}^{T-1} \left( \frac{1+\rho P_{s+1}^M}{P_s^M} \frac{P_s}{P_{s+1}} \right)$  for  $T \geq 1$  and  $R_{t,t}^M = 1$ , see [Preston and Eusepi \(2011\)](#). This solvency constraint means that Ricardian agents do not overaccumulate debt, and must hold with equality in equilibrium.

First-order conditions (FOCs) associated with Ricardian agents' optimization problem are given by:

$$P_t^M = \beta \mathbb{E}_t \left\{ \left( \frac{C_t^o}{C_{t+1}^o} \right)^\sigma \frac{1}{\Pi_{t+1}} (1 + \rho P_{t+1}^M) \right\}, \quad (5)$$

$$(1 - \tau_t) \left( \frac{W_t}{P_t} \right) = (N_t^o)^\varphi (C_t^o)^\sigma, \quad (6)$$

where  $\Pi_{t+1} \equiv P_t/P_{t+1}$  is the gross inflation rate.

For later use, the stochastic discount factor is defined as:

$$Q_{t,t+1} \equiv \beta \left( \frac{C_t^o}{C_{t+1}^o} \right)^\sigma \left( \frac{P_t}{P_{t+1}} \right),$$

where  $\mathbb{E}_t [Q_{t,t+1}] = R_t^{-1}$  is the inverse of the short-term nominal interest rate,  $R_t$ , which is the monetary policy's instrument.

Equation (5) gives the optimal allocation across time, and prices the declining payoff consols. If  $\rho = 0$ , the model reduces to the single period bonds case, and the price of these bonds will be given by  $P_t^M = R_t^{-1}$ . Otherwise, the presence of long term bonds introduces the term structure of interest rates to the model.

The latter optimality condition (6) states that the marginal rate of substitution between consumption and leisure equals the after-tax real wage rate.

## Liquidity constrained households

Rule-of-thumb consumers are unable to smooth their consumption path over time. At each period they solve a static problem, i.e., they maximize their period utility:

$$\frac{(C_t^r)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^r)^{1+\varphi}}{1+\varphi}, \quad (7)$$

subject to the constraint that all of their labor income net of taxes is consumed:

$$P_t C_t^r = (1 - \tau_t) W_t N_t^r. \quad (8)$$

The associated first-order condition yields:

$$(1 - \tau_t) \frac{W_t}{P_t} = (N_t^r)^\varphi (C_t^r)^\sigma. \quad (9)$$

Note from the liquidity constrained consumers' problem that they do not intertemporally substitute consumption in response to changes in interest rates.

As in [Albonico and Rossi \(2017\)](#), firms are indifferent with respect to the type of agent they employ. Therefore, the equilibrium on the competitive labor market requires:

$$(N_t^o)^\varphi (C_t^o)^\sigma = (N_t^r)^\varphi (C_t^r)^\sigma, \quad (10)$$

that is, the marginal rates of substitution between consumption and leisure of Ricardian and liquidity constrained consumers are equalized.

### Aggregation

Aggregate private consumption and labor supply are defined as follows:

$$C_t \equiv \lambda C_t^r + (1 - \lambda) C_t^o, \quad (11)$$

$$N_t \equiv \lambda N_t^r + (1 - \lambda) N_t^o. \quad (12)$$

### 3.2 Firms and price-setting

There is a continuum of monopolistically competitive firms, indexed by  $j \in [0, 1]$ , producing differentiated intermediate goods. A typical firm in the intermediate good sector produces a differentiated consumption good subject to a linear production function:

$$Y_t(j) = N_t(j). \quad (13)$$

Each firm faces a demand curve for their product that is given by:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_t} Y_t, \quad (14)$$

where  $Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj \right]^{\frac{\epsilon_t}{\epsilon_t-1}}$  and  $P_t = \left[ \int_0^1 P_t(j)^{1-\epsilon_t} dj \right]^{\frac{1}{1-\epsilon_t}}$ . The elasticity of substitution between varieties,  $\epsilon_t$ , is assumed to be a time-varying exogenous AR(1) process given by:

$$\ln(\epsilon_t) = (1 - \rho_\epsilon) \ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, 1), \quad (15)$$

where  $0 \leq \rho_\epsilon < 1$ . The assumption of a stochastic elasticity of substitution allows for variations in the desired price markups and, hence, it is a device to introduce markup shocks to the model, as in [Beetsma and Jensen \(2004\)](#).

Following [Rotemberg \(1982\)](#), sluggish price adjustment is introduced by assuming that firms face a resource cost for adjusting their nominal prices that is quadratic in price changes and proportional to the nominal level of activity. This quadratic cost is defined for a monopolistic firm  $j$  as<sup>7</sup>:

$$\eta_t(j) \equiv \frac{\phi}{2} \left( \frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1 \right)^2 Y_t, \quad (16)$$

where  $\phi \geq 0$  measures the degree of nominal price stickiness and  $\Pi^*$  is the chosen inflation target.

<sup>7</sup>We consider the [Rotemberg \(1982\)](#) pricing approach, rather than [Calvo \(1983\)](#) pricing, because this reduces the number of endogenous state variables. In the latter, price dispersion becomes an additional endogenous state variable, which complicates matters when solving the model through nonlinear methods. However, as [Leith and Liu \(2016\)](#) and [Sims and Wolff \(2017\)](#) conclude, the form of nominal inertia adopted is not innocuous for higher order of approximations.



The problem facing firm  $j$  is to maximize the discounted value of nominal profits:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{z=0}^{\infty} Q_{t,t+z} \Xi_{t+z}(j),$$

where nominal profits are defined as,

$$\Xi_t(j) \equiv P_t(j)Y_t(j) - mc_t Y_t(j)P_t - \frac{\phi}{2} \left( \frac{P_t(j)}{\Pi^* P_{t-1}(j)} - 1 \right)^2 P_t Y_t,$$

subject to the linear production function (13), the demand schedule for their product (14), and the quadratic adjustment costs in changing prices (16).

The first-order condition for a symmetric equilibrium implies the following Rotemberg version of the nonlinear New Keynesian Phillips curve (NKPC):

$$\frac{\Pi_t}{\Pi^*} \left( \frac{\Pi_t}{\Pi^*} - 1 \right) = \beta \mathbb{E}_t \left[ \left( \frac{C_t^o}{C_{t+1}^o} \right)^\sigma \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi^*} \left( \frac{\Pi_{t+1}}{\Pi^*} - 1 \right) \right] + \phi^{-1} ((1 - \epsilon_t) + \epsilon_t mc_t), \quad (17)$$

relating current inflation to future expected inflation and to the level of activity.

Defining the real marginal cost of production as  $mc_t \equiv W_t/P_t$  and combining it with equations (6) and (9), this expression can be rewritten as  $mc_t = (1 - \tau_t)^{-1} [\lambda (N_t^r)^\varphi (C_t^r)^\sigma + (1 - \lambda) (N_t^o)^\varphi (C_t^o)^\sigma]$ .

### 3.3 Government

The government comprises a single policymaker who coordinates monetary and fiscal policies aiming to maximize welfare of both Ricardian and liquidity constrained consumers. Monetary policy uses nominal interest rate on short-term nominally riskless discount bonds,  $R_t$ , as its instrument. Fiscal policy's control variables are the level of government consumption,  $G_t$ , and distortionary labor income taxes,  $\tau_t$ .

The level of aggregate government expenditures on the provision of public goods takes the same form as private consumption:

$$G_t = \left( \int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

such that government's demand for individual goods is given by:

$$G_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_t} G_t.$$

Government expenditures are financed by levying distortionary taxation on labor income at the rate  $\tau_t$ , and by issuing long-term bonds,  $B_t^M$ . The government solvency constraint can then be written, in real terms, as:

$$P_t^M b_t = (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t, \quad (18)$$

where  $b_t \equiv B_t^M/P_t$  denotes real debt and  $w_t \equiv W_t/P_t$  are real wages.

The nominal nature of debt widens the interaction between monetary and fiscal policies. Monetary nominal interest rate policy decisions can affect the government budget in the following ways: (i) influencing directly the nominal return on government's instruments; (ii) an indirect effect on the real market value of outstanding government debt, via changes in the price

level; and (iii) on a sticky-price environment, the real effects of monetary policy can change the size of the tax base (Leeper et al., 2016).

The maturity of the government debt also plays an important role. If all government debt is comprised of one-period debt, that is  $\rho = 0$ , by equation (18) adjustments on the ex-post real return on bonds are only possible through changes in current period inflation  $\Pi_t$ , which can be costly in a sticky-price economy. A longer maturity of government debt,  $0 < \rho < 1$ , introduces another source of adjustments in the ex-post real return. Changes in the bond price,  $P_t^M$ , which depends on future inflation, can help to achieve the necessary adjustments on real debt return at a smaller cost.

### 3.4 Market Clearing

Goods market clearing requires, for each good  $j$ ,

$$Y_t(j) = C_t(j) + G_t(j) + \eta_t(j),$$

such that, in a symmetrical equilibrium,

$$Y_t \left[ 1 - \frac{\phi}{2} \left( \frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right] = C_t + G_t. \quad (19)$$

Alongside, market clearing condition in the bonds market requires that the portfolio of long-term bonds held by households evolves according to the government's budget constraint.

Before analyzing the optimal time-consistent policy problem, the competitive rational expectations equilibrium is defined as follows:

**Definition 1 (Competitive equilibrium).** A competitive rational expectations equilibrium consists of a plan  $\{C_t^o, C_t^r, N_t^o, N_t^r, \Pi_t\}_{t=0}^\infty$  satisfying: (i) households' budget constraints (3) and (8); (ii) optimality conditions (5), (6), (9) and (17); (iii) the production function  $Y_t = N_t$ , and aggregations (11) and (12); (iv) equilibrium in the competitive labor market (10); (v) the government's budget constraint (18); (vi) the market clearing condition (19); and (vii) the no-Ponzi-game condition (4), given the government policies  $\{R_t, G_t, \tau_t, b_t\}_{t=0}^\infty$ , prices  $\{w_t, P_t^M\}_{t=0}^\infty$ , the exogenous process  $\{\epsilon_t\}_{t=0}^\infty$  and an initial level of government debt  $b_{t-1}$ .

## 4 Joint optimal time-consistent policy

Fiscal and monetary policies are assumed to be conducted by a single policymaker constrained to act in a time-consistent manner. Acting under discretion, the policymaker is unable to commit to any particular future plan and, instead, reoptimizes his responses at each period. However, the presence of an endogenous state variable in the form of government debt yields the optimal discretionary policy history dependent and, hence, policy actions made today can have effects on future expectations through the debt stock that the policy bequeaths to the future.

Following Albonico and Rossi (2017), we assume that the welfare criterion adopted by the policymaker is to maximize a weighted average of Ricardian and liquidity constrained consumers' utilities. That is, the policy under discretion is described by a set of decision rules  $\{C_t^o, C_t^r, N_t^o, N_t^r, Y_t, \Pi_t, b_t, \tau_t, G_t\}$  which maximize the value function:

$$V(b_{t-1}, \epsilon_t) = \max \{ \lambda u(C_t^r, G_t, N_t^r) + (1 - \lambda) u(C_t^o, G_t, N_t^o) + (\delta\beta) \mathbb{E}_t[V(b_t, \epsilon_{t+1})] \}, \quad (20)$$

subject to the budget constraint of rule-of-thumb consumers (8), the equalization condition of marginal utilities between Ricardian and liquidity constrained consumers (10), aggregate

conditions (11)-(12), the production function (13), the New Keynesian Phillips curve (17), the resource constraint (19), and the government's budget constraint (18). The parameter  $0 \leq \delta \leq 1$  captures the possibility that the policymaker may suffer from a degree of myopia, that is, it can discount the future more heavily than society does<sup>8</sup>.

Defining the following state-dependent auxiliary functions to capture future expectations:

$$\begin{aligned} M(b_t, \epsilon_{t+1}) &\equiv (C_{t+1}^o)^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*} \left( \frac{\Pi_{t+1}}{\Pi^*} - 1 \right), \\ L(b_t, \epsilon_{t+1}) &\equiv (C_{t+1}^o)^{-\sigma} (\Pi_{t+1})^{-1} (1 + \rho P_{t+1}^M), \end{aligned}$$

the Lagrangian for the policy problem can, after some algebraic manipulations, be written as:

$$\begin{aligned} \mathcal{L} = & \left\{ \lambda \left[ \frac{(C_t^r)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^r)^{1+\varphi}}{1+\varphi} \right] + (1-\lambda) \left[ \frac{(C_t^o)^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \right] + (\delta\beta) \mathbb{E}_t[V(b_t, \epsilon_{t+1})] \right\} \\ & + \mu_{1t} \left[ Y_t \left( 1 - \frac{\phi}{2} \left( \frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right) - \lambda C_t^r - (1-\lambda) C_t^o - G_t \right] \\ & + \mu_{2t} \left[ (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} [(N_t^o)^\varphi (C_t^o)^\sigma] - \phi \frac{\Pi_t}{\Pi^*} \left( \frac{\Pi_t}{\Pi^*} - 1 \right) + \beta \phi (C_t^o)^\sigma Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] \right] \\ & + \mu_{3t} \left[ \beta (C_t^o)^\sigma b_t \mathbb{E}_t[L(b_t, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t} (1 + \rho \beta (C_t^o)^\sigma \mathbb{E}_t[L(b_t, \epsilon_{t+1})]) + \left( \frac{\tau_t}{1-\tau_t} \right) [(N_t^o)^\varphi (C_t^o)^\sigma] Y_t - G_t \right] \quad (21) \\ & + \mu_{4t} [Y_t - \lambda N_t^r - (1-\lambda) N_t^o] \\ & + \mu_{5t} [(N_t^r)^\varphi (C_t^r)^\sigma - (N_t^o)^\varphi (C_t^o)^\sigma] \\ & + \mu_{6t} [C_t^r - (N_t^r)^{1+\varphi} (C_t^r)^\sigma] \end{aligned}$$

where the bond pricing equation (5) was used to eliminate the current value of the bond in the government's budget constraint. As [Leeper et al. \(2019\)](#) emphasize, although the policymaker optimizes with respect to all endogenous variables, it is not acting as a social planner. Instead, it is influencing the decentralized equilibrium by choosing its policy instruments, aiming to maximize the objective function subject to the time-consistency constraint.

The implied set of first-order conditions is given by:

$$\begin{aligned} C_t^o : & \quad (1-\lambda)(C_t^o)^{-\sigma} - \mu_{1t}(1-\lambda) + \sigma\mu_{2t} [\epsilon_t(1-\tau_t)^{-1} (N_t^o)^\varphi (C_t^o)^{\sigma-1} + \beta\phi(C_t^o)^{\sigma-1} Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})]] \\ & \quad + \sigma\mu_{3t} \left[ \beta(C_t^o)^{\sigma-1} b_t \mathbb{E}_t[L(b_t, \epsilon_{t+1})] - \rho\beta \frac{b_{t-1}}{\Pi_t} (C_t^o)^{\sigma-1} \mathbb{E}_t[L(b_t, \epsilon_{t+1})] + \left( \frac{\tau_t}{1-\tau_t} \right) (N_t^o)^\varphi (C_t^o)^{\sigma-1} Y_t \right] \\ & \quad - \sigma\mu_{5t} (N_t^o)^\varphi (C_t^o)^{\sigma-1} = 0, \\ C_t^r : & \quad \lambda(C_t^r)^{-\sigma} - \lambda\mu_{1t} + \sigma\mu_{5t} (N_t^r)^\varphi (C_t^r)^{\sigma-1} + \mu_{6t} [1 - \sigma(N_t^r)^{1+\varphi} (C_t^r)^{\sigma-1}] = 0, \\ N_t^o : & \quad (\lambda-1)(N_t^o)^\varphi + \varphi\mu_{2t}\epsilon_t(1-\tau_t)^{-1} (N_t^o)^{\varphi-1} (C_t^o)^\sigma + \varphi\mu_{3t} \left( \frac{\tau_t}{1-\tau_t} \right) (N_t^o)^{\varphi-1} (C_t^o)^\sigma Y_t \\ & \quad - (1-\lambda)\mu_{4t} - \varphi\mu_{5t} (N_t^o)^{\varphi-1} (C_t^o)^\sigma = 0, \\ N_t^r : & \quad \lambda(N_t^r)^\varphi + \lambda\mu_{4t} - \varphi\mu_{5t} (N_t^r)^{\varphi-1} (C_t^r)^\sigma + (1+\varphi)\mu_{6t} (N_t^r)^\varphi (C_t^r)^\sigma = 0, \\ G_t : & \quad \chi G_t^{-\sigma_g} - \mu_{1t} - \mu_{3t} = 0, \\ Y_t : & \quad \mu_{1t} \left( 1 - \frac{\phi}{2} \left( \frac{\Pi_t}{\Pi^*} - 1 \right)^2 \right) - \beta\phi\mu_{2t} (C_t^o)^\sigma Y_t^{-2} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] + \mu_{3t} \left( \frac{\tau_t}{1-\tau_t} \right) (N_t^o)^\varphi (C_t^o)^\sigma + \mu_{4t} = 0, \\ \Pi_t : & \quad -\phi\mu_{1t} \frac{Y_t}{\Pi^*} \left( \frac{\Pi_t}{\Pi^*} - 1 \right) - \mu_{2t} \frac{\phi}{\Pi^*} \left( 2 \frac{\Pi_t}{\Pi^*} - 1 \right) + \mu_{3t} \frac{b_{t-1}}{\Pi_t^2} (1 + \rho\beta(C_t^o)^\sigma \mathbb{E}_t[L(b_t, \epsilon_{t+1})]) = 0, \\ \tau_t : & \quad \mu_{2t}\epsilon_t + \mu_{3t}Y_t = 0, \\ b_t : & \quad -(\delta\beta) \mathbb{E}_t \left[ \mu_{3t+1} \frac{1}{\Pi_{t+1}} (1 + \rho P_{t+1}^M) \right] + \beta\phi\mu_{2t} (C_t^o)^\sigma Y_t^{-1} \mathbb{E}_t[M(b_t, \epsilon_{t+1})] \\ & \quad + \mu_{3t} \left[ \beta(C_t^o)^\sigma \mathbb{E}_t[L(b_t, \epsilon_{t+1})] + \beta(C_t^o)^\sigma b_t \mathbb{E}_t[L(b_t, \epsilon_{t+1})] - \rho\beta \frac{b_{t-1}}{\Pi_t} (C_t^o)^\sigma \mathbb{E}_t[L(b_t, \epsilon_{t+1})] \right] = 0, \end{aligned}$$

<sup>8</sup>The parameter  $\delta$  can be interpreted as an exogenous probability of the policymaker being voted out of office in the following period ([Stehn and Vines, 2008](#)).

where  $X_b(b_t, \epsilon_{t+1}) \equiv \partial X(b_t, \epsilon_{t+1})/\partial b_t$  for functions  $X \in \{L, M\}$ , and we have used the envelope theorem on the first-order condition for government debt to obtain:

$$\frac{\partial V(b_{t-1}, \epsilon_t)}{\partial b_{t-1}} = -\mu_{3t} \frac{1}{\Pi_t} (1 + \rho P_t^M).$$

Optimality condition for inflation reveals another possible source of inflationary bias, besides the standard bias associated with discretionary policies. The first term captures the costs, in terms of the resource constraint, of raising inflation, while the second highlights the positive effects on the output-inflation trade-off, given expectations, in the New Keynesian Phillips curve when the economy is at a suboptimal position. The last term in the first-order condition captures another benefit of raising inflation, the negative effect it has on the real market value of government debt. Ricardian agents will perceive that a higher debt is an incentive for government to introduce inflation surprises in an attempt to reduce the debt burden, in doing so they are expected to raise their inflationary expectations,  $\mathbb{E}_t[M_b(b_t, \epsilon_{t+1})] > 0$ , until that incentive is offset. [Leith and Wren-Lewis \(2013\)](#) labeled this the “debt stabilization bias”.

The first-order condition for debt can be rewritten as:

$$P_t^M \mu_{3t} - (\delta\beta)\mathbb{E}_t \left[ \frac{\mu_{3t+1}}{\Pi_{t+1}} (1 + \rho P_{t+1}^M) \right] - \mu_{3t} \left\{ \phi\beta\epsilon^{-1}\mathbb{E}_t[M_b(b_t, \epsilon_{t+1})] - \left[ \left( b_t - \rho \frac{b_{t-1}}{\Pi_t} \right) \mathbb{E}_t[L_b(b_t, \epsilon_{t+1})] \right] \right\} = 0, \quad (22)$$

the first two terms of the equation is a version of the standard tax-smoothing argument of [Barro \(1979\)](#), requiring that marginal costs of taxation are smoothed over time. Under commitment, only these two terms would appear, implying that the steady state of debt will follow a random walk ([Leeper et al., 2016](#)).

Given that the policymaker is constrained to act in a time-consistent manner, its decisions can affect future variables through the level of debt it bequeaths. This is captured by the partial derivatives of debt in the last term of equation (22)<sup>9</sup>. As in [Leeper et al. \(2019\)](#), the numerical results obtained in this work yield: (i)  $M_b(b_t, \epsilon_{t+1}) > 0$ , which means that, as previously discussed, inflation expectations rise when debt levels are higher. Hence, with nominal rigidity, the policymaker has an incentive to reduce debt and inflation, deviating from tax smoothing; and (ii)  $L_b(b_t, \epsilon_{t+1}) < 0$ , implying a negative relationship between debt level and bond prices. A higher level of debt leads to lower bond prices, given that it raises inflation. With lower prices on government bonds, the policymaker needs to issue more bonds to finance its debt, but at the same time it also means that it needs to pay less to buy back the existing debt stock. As the maturity of the debt increases, the latter effect rises relative to the former, and may even be reversed, reducing the incentive to lower debt levels that the policymaker is facing.

The nonlinear system of first-order conditions previously described seems to be largely unaffected by the fraction of liquidity constrained consumers in the economy. However, this apparent invariance is deceptive. Increases in the share of rule-of-thumb consumers,  $\lambda$ , are, ceteris paribus, associated with higher per capita profits in the sector of monopolistically competitive firms, increasing the monopolistic distortion ([Albonico and Rossi, 2017](#)). Therefore, the presence of liquidity constrained agents constitutes an additional distortion in the economy. These distortions, alongside the degree of myopia of the policymaker and the maturity of debt it issues, as we shall see in the numerical results, are crucial in determining the long run equilibrium rate of both inflation and debt-to-GDP ratio.

<sup>9</sup>The presence of partial derivatives of policy functions makes this relation a generalized Euler equation - as labeled by [Krusell et al. \(2002\)](#) - which needs to be solved numerically, given that, in general, the form of these functions are unknown.

## 5 Numerical methods and calibration

This section outlines the numerical method used to solve for the discretionary equilibrium and the calibration of parameters.

### 5.1 Solution method

For the model described in sections 3 and 4, the equilibrium policy functions cannot be computed analytically and, thus, numerical methods are necessary. Linearization around the steady state, a common approach in economics, is not feasible since the presence of a Generalized Euler Equation yields that steady state endogenous and, therefore, *a priori* unknown. Following [Leeper et al. \(2016, 2019\)](#), we resort on a global approximation method to solve for the time consistent equilibrium of the model<sup>10</sup>. More specifically, the policy functions are approximated by Chebyshev polynomials and the nonlinear system of equations is iterated until a set of time-invariant equilibrium of policy rules mapping the vector of state variables to the optimal decisions is reached<sup>11</sup>. The numerical algorithm is detailed in Appendix B.

### 5.2 Calibration

The model is calibrated to a quarterly frequency. The baseline parameterization is summarized on Table 1, which is in line with [Leeper et al. \(2016\)](#); [Albonico and Rossi \(2017\)](#). The discount factor for Ricardian households is set to 0.995, implying an annual real interest rate of 2%. The intertemporal elasticity of substitution between private consumption,  $\sigma$ , and public consumption,  $\sigma_g$ , are equal to one half ( $\sigma = \sigma_g = 2$ ). The baseline value of  $\varphi$  is set to be consistent with a Frisch elasticity of labor supply of one-third (i.e.,  $\varphi = 3$ ). The elasticity of substitution between varieties is assumed to be  $\bar{\epsilon} = 21$ , to be consistent with a markup of 5% as in [Siu \(2004\)](#). The scaling parameter  $\chi = 0.055$  is calibrated to ensure that, in the steady state, the government spending-to-GDP ratio,  $G/Y$ , is approximately 19%. The annual inflation target,  $\Pi^*$ , is assumed to be 2%, a value which is in line with the adopted by most inflation targeting economies. The coupon decay parameter  $\rho$  is calibrated to 0.9598 to ensure that, given the discount factor of Ricardian households  $\beta$  and the inflation target  $\Pi^*$ , the average term to maturity of debt corresponds to 5 years, a duration compatible with data from most OECD countries (see [Eusepi and Preston \(2013\)](#)). The Rotemberg price adjustment cost parameter,  $\phi = 32.5$ , implies that on average firms re-optimize prices approximately every six months<sup>12</sup> - an empirically plausible value. The degree of myopia of the policymaker is set to  $\delta = 0.9899$ , which implies a time horizon of approximately 25 years. The parameters characterizing the cost-push exogenous process are given by  $\rho_\epsilon = 0.95$  and  $\sigma_\epsilon = 0.01$ .

Following [Albonico and Rossi \(2017\)](#), the fraction of liquidity constrained consumers can assume three alternative values:  $\lambda \in \{0; 0.3; 0.5\}$ . When  $\lambda = 0$ , all consumers of the model are optimizing agents and, hence, the model is the standard representative agent New Keynesian model augmented to include the government's budget constraint. The empirical literature on limited asset market participation reports that the fraction of rule-of-thumb consumers lies

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<sup>10</sup>Time consistency problems, in general, can be treated as dynamic games. In problems of this kind, multiplicity of equilibria often arises. By using polynomial approximations, we are focusing only on continuous equilibria. See [Judd \(2004\)](#) for a discussion on the existence, uniqueness and alternative computational approaches to deal with problems of this kind.

<sup>11</sup>For textbook treatments on the numerical techniques involved see [Judd \(1998\)](#); [Miranda and Fackler \(2004\)](#); [Fernández-Villaverde et al. \(2016\)](#).

<sup>12</sup>Given the equivalence between Calvo and Rotemberg pricing for linearized models,  $\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$ , where  $\theta$  is the fraction of firms that keep their prices unchanged on Calvo model, see [Leith and Liu \(2016\)](#); [Sims and Wolf \(2017\)](#).

Parameter	Value	Description
$\beta$	0.995	Quarterly discount factor (household)
$\delta$	0.9899	Degree of myopia (policymaker)
$\sigma$	2	Relative risk aversion coefficient
$\sigma_g$	2	Relative risk aversion coefficient (government spending)
$\varphi$	3	Inverse Frisch elasticity of labor supply
$\chi$	0.055	Scaling parameter (government spending)
$\lambda$	varying	Fraction of liquidity constrained consumers
$\rho$	0.9598	Debt maturity structure (5 years)
$\Pi^*$	2%	Annual inflation target
$\phi$	32.5	Rotemberg adjustment cost coefficient
$\bar{\epsilon}$	21	Elasticity of substitution between varieties
$\rho_\epsilon$	0.95	AR-coefficient of cost-push shock
$\sigma_\epsilon$	0.01	Standard deviation of cost-push shock

Table 1: Calibration

between 30% and 50% of the agents population. [Campbell and Mankiw \(1989\)](#); [Muscatelli et al. \(2004\)](#) estimated values for  $\lambda$  in the neighborhood of 50%, the empirical microeconomic evidence surveyed in [Mankiw \(2000\)](#) is in line with those findings. [Di Bartolomeo et al. \(2011\)](#) found that, on average, the fraction of non-Ricardian agents for the G7 countries is 26%. More recently, [Albonico et al. \(2014, 2016\)](#) report values of the share of liquidity constrained consumers in between 25% and 53% for the Euro area, and 47% for the US. Those findings motivate our choice for the values of  $\lambda$ .

## 6 Numerical results

This section reports the numerical results obtained in the solution of the optimal discretionary policy formerly described. Subsection 6.1 explores the long-run equilibrium properties under different parameterizations. Optimal dynamics in response to a markup shock through impulse response functions are considered in Subsection 6.2. The role of the maturity structure of the government debt in the presence of limited asset markets participation is discussed in Subsection 6.3. Lastly, in Subsection 6.4 introduces the welfare effects of LAMP and higher levels of public debt.

### 6.1 Steady State

Table 2 summarizes the steady state values under different parameterizations of the model. Under the benchmark calibration, when all households on the economy are Ricardian agents, the combination of a myopic policymaker alongside a government debt with a maturity structure of five years<sup>13</sup> implies a positive debt-to-GDP ratio,  $\frac{bP^M}{4Y}$ , of 55.6% and an annualized rate of inflation in the steady state of 5.76%, which outweighs the established target of 2%. The magnitude of the inflationary bias, that causes the overshooting of inflation with respect to the target, is determined by an inefficiently low equilibrium level of output that gives the

<sup>13</sup>When the discount factor of both policymaker and households are equal and/or debt is short-term, numerical results show that the steady state debt-to-GDP ratio is negative, which means that the government accumulates assets rather than issues liabilities. This is in line with much of the literature, see the discussion on Section 2 and [Debortoli and Nunes \(2013\)](#); [Leeper et al. \(2016, 2019\)](#).

discretionary policymaker an incentive to generate surprise inflation to push output closer to the efficient steady state level. In standard analyses of the inflationary bias problem, the solely source of inefficiency is the degree of monopolistic competition that distorts the steady state. However, in the presence of government debt and distortionary taxation, this inflationary bias problem becomes state-dependent, since at higher levels of debt or taxes, the inefficiency is accentuated and, therefore, increasing the desire to induce surprise inflation.

Variable	Benchmark calibration			$\phi = 50$			$\frac{\bar{\epsilon}-1}{\epsilon} = 6\%$		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
$\frac{bPM}{4Y}$	55.6%	12.6%	-9.0%	64.9%	7.8%	-22.6%	40.4%	-3.6%	-26.4%
$(\Pi^4 - 1)$	5.76%	4.88%	4.45%	5.29%	4.44%	3.97%	5.52%	4.59%	3.88%
$(R^4 - 1)$	7.90%	7.01%	6.57%	7.42%	6.55%	6.08%	7.65%	6.70%	5.98%
$Y$	1.029	1.031	1.033	1.028	1.032	1.034	1.027	1.030	1.031
$G/Y$	19.3%	19.4%	19.4%	19.3%	19.4%	19.5%	19.3%	19.4%	19.5%
$\tau$	21.4%	20.6%	20.2%	21.6%	20.5%	20.0%	21.4%	20.5%	20.1%
$C^o$	0.829	0.844	0.860	0.829	0.844	0.859	0.827	0.844	0.862
$C^r$	-	0.800	0.803	-	0.800	0.805	-	0.794	0.798
$C$	0.829	0.831	0.832	0.829	0.831	0.832	0.827	0.829	0.830
$N^o$	1.029	1.020	1.009	1.028	1.021	1.011	1.027	1.017	1.005
$N^r$	-	1.058	1.056	-	1.057	1.056	-	1.059	1.058
$\frac{\Xi}{1-\lambda}$	0.048	0.069	0.097	0.047	0.069	0.097	0.057	0.082	0.116
$V^o$	-352.7	-346.2	-339.2	-352.8	-346.3	-339.9	-352.8	-345.5	-337.6
$V^r$	-	-367.7	-366.2	-	-367.4	-365.3	-	-369.7	-368.1
$V$	-352.7	-352.6	-352.7	-352.8	-352.6	-352.6	-352.8	-352.7	-352.9

Table 2: Steady state: Benchmark calibration, price flexibility and monopolistic competition.

The presence of liquidity constrained consumers constitutes another source of distortion to the model economy. As [Albonico and Rossi \(2017\)](#) pointed out and our numerical results show, when the fraction of rule-of-thumb agents,  $\lambda$ , increases, per capita profits earned by optimizing agents,  $\frac{\Xi}{1-\lambda}$ , also rises<sup>14</sup>, pushing the monopolistic distortion even further. A more pronounced monopolistic distortion strengthens the state-dependent inflationary bias, for a given level of debt. Optimizing agents, aware of this higher inflationary bias, raise their inflationary expectations accordingly. As a result, the debt stabilization bias also increases - the discretionary policymaker has now a stronger incentive to influence the endogenous inflationary bias and to mitigate the costs of distortionary taxation through reductions in the level of debt. Therefore, increases in the share of rule-of-thumb consumers on the economy are associated with substantial decreases in both the steady state rate of inflation and debt-to-GDP ratio - the latter even turns negative when  $\lambda = 0.5$ . Since government spending-to-GDP ratio,  $G/Y$ , is mainly unresponsive to variations in  $\lambda$ , lower levels of debt can be financed by lower tax rates. Other model variables are changed only marginally.

As a departure from the benchmark calibration, we consider firstly an increase in the rigidity of prices,  $\phi$ , meaning that inflation is now more costly and that the efficacy of monetary policy in affecting the real side of the economy is enhanced. Consequently, the policymaker has now a stronger incentive to reduce the state-dependent inflationary bias problem. As a result, in general, the equilibrium steady state rate of inflation and debt-to-GDP ratio are lower when

<sup>14</sup>This increase in the financial wealth of optimizing agents ends up boosting Ricardian consumption in the steady state and, thus, has distributional consequences, see Subsection 6.4.

prices are less flexible. The Ricardian agents economy,  $\lambda = 0$ , highlights the fact that equilibrium values are highly dependent on both the magnitude of the government debt stock and its maturity. Under the benchmark calibration, the policymaker faces a high level of debt stock. Once the rigidity of prices rises, the incentive to boost activity through inflation lessens (the inflation bias problem is reduced). Moreover, now it is more costly for the policymaker to use current inflation as a device to reduce the debt burden (steady state equilibrium inflation rate reduces to 5.29%). Therefore, due to the mix of a long-term maturity alongside a high level of debt stock, the steady state debt-to-GDP ratio rises instead - to 64.9% - as this lowers bond prices and, thus, makes it more cheaper for the government to buy back the existing debt stock<sup>15</sup>.

Lastly, a more pronounced monopolistic distortion is considered by increasing the markup by one percentage point. Under a greater monopolistic distortion, the inefficiencies of the economy are higher and, hence, the inflationary bias problem is also higher for a given level of debt. As a consequence of that, the policymaker faces a greater incentive to influence the endogenous inflationary bias by reducing the level of debt. By raising the debt stabilization bias, the government ensures that the debt-to-GDP ratio and the inflation rate of equilibrium are lower.

To summarize, the effects that the existence of limited asset market participation have on the jointly optimal monetary and fiscal policy are robust to different parameterization scenarios. Increases in the share of rule-of-thumb consumers, by raising the per capita financial wealth of optimizing agents, deepens the degree of monopolistic distortions in the economy. In doing so, strengthens both the state-dependent inflationary and debt stabilization biases leading to a reduction in the steady state values of debt-to-GDP ratio and inflation rate. Therefore, the fraction of liquidity consumers in the economy is a key driver of the equilibrium rate of inflation and debt stock, alongside the debt maturity and the degrees of myopia of the policymaker<sup>16</sup> and monopolistic competition (Leeper et al., 2019). It is important to note that, while the overall value function,  $V$ , is largely unaffected by the share of rule-of-thumb agents, the same is not true for their individual counterparts -  $V^o$  and  $V^r$  - suggesting that the degree of access to asset markets can have distributional consequences (see Subsection 6.4).

## 6.2 Optimal dynamics in response to a markup shock

Figure 1 displays the optimal dynamics of the key macroeconomic variables in response to a positive markup shock through impulse response functions (IRFs henceforth). All variables are measured as percentage deviation from the steady state. Under the benchmark calibration, we consider the cases in which all agents are Ricardians ( $\lambda = 0$ ) and when the fraction of liquidity constrained consumers is  $\lambda = 0.3$  and  $\lambda = 0.5$ <sup>17</sup>.

<sup>15</sup>To test the robustness of these results, we considered the following alternative parameterizations of the model: (i) single-period debt ( $\rho = 0$ ); and (ii) a low degree of myopia ( $\delta = 0.995$ ) - which implies a lower level of debt in the steady state. An increase in the sluggishness of prices causes steady debt-to-GDP ratio falls from 10.8% to 9% in the single-period scenario, and from 4.7% to 2.5% when the policymaker is more patient. Inflation rate also falls: from 3.8% to 3.6% when  $\rho = 0$ , and from 3.7% to 3.5% when  $\delta = 0.995$ . This corroborates the results obtained.

<sup>16</sup>The roles of debt maturity and degree of myopia are discussed, respectively, in Subsections 6.3 and 6.4.

<sup>17</sup>It is important to note that all cases considered are associated with a different level of government debt in the steady state and that those different conditions can have different implications to the optimal dynamics.



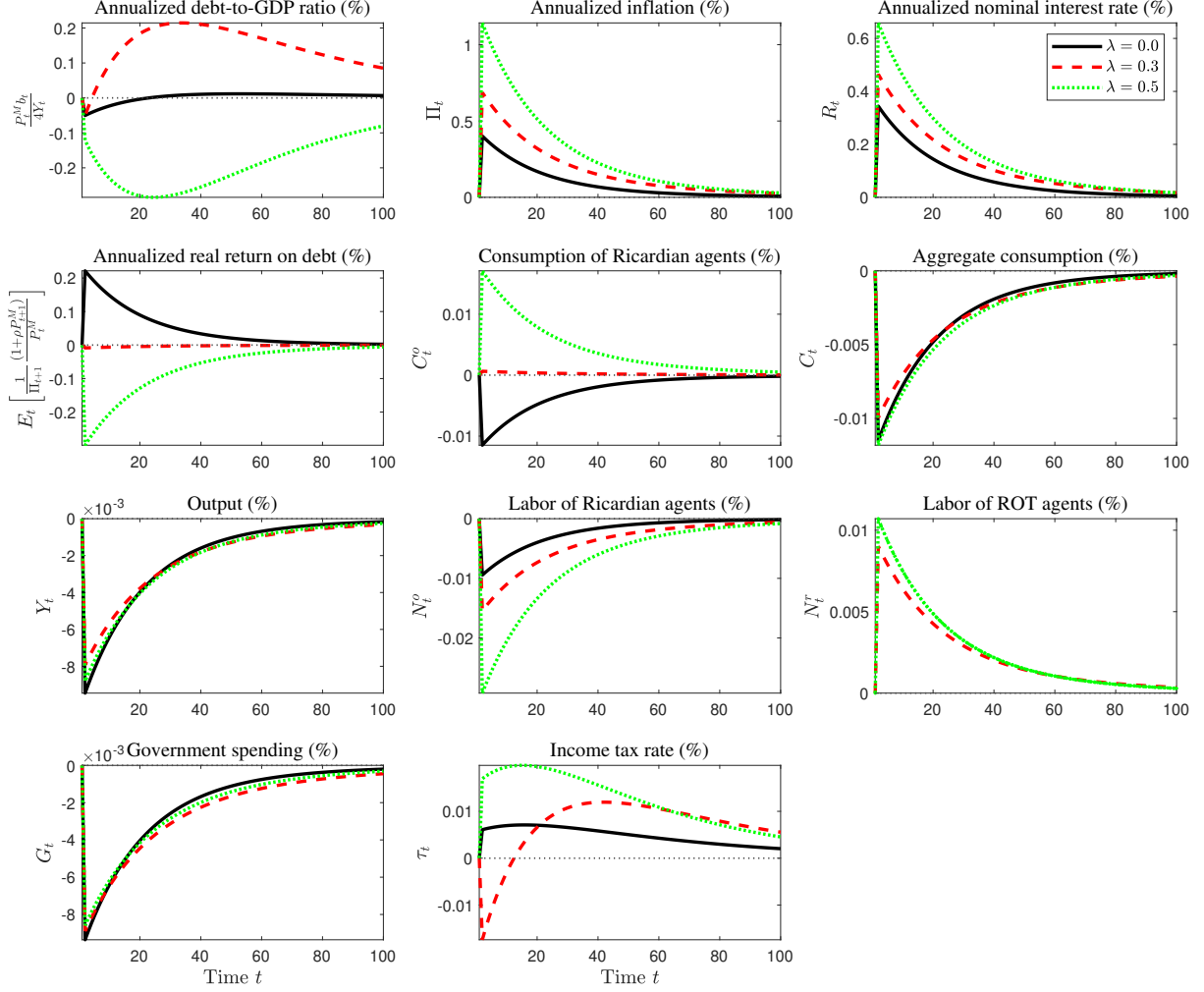


Figure 1: IRFs to a positive markup shock under the benchmark calibration with  $\lambda = 0$  (black solid line),  $\lambda = 0.3$  (red dashed lines) and  $\lambda = 0.5$  (green dotted lines). All variables measured as percentage deviation from steady state.

Before turning to the analysis of IRFs for each scenario, it is convenient to make some general observations. The existence of real imperfections in the form a cost-push shock in our model economy breaks the ‘divine coincidence’ and, thus, the optimizing discretionary policymaker faces a policy tradeoff between stabilization of inflation and output. In all the cases considered, following a decrease of one standard deviation,  $\sigma_\epsilon$ , in the elasticity of substitution between intermediate goods, which implies an increase in firms markup, the optimal response made by the policymaker acting time-consistently is to accommodate only partially the inflationary consequences of the shock and, in doing so, allowing a fall in output.

**Ricardian agents economy.** When all households can smooth consumption intertemporally,  $\lambda = 0$ , the optimal dynamics in response to the markup shock are mainly driven by a desire to reduce debt levels through higher distortionary wage income tax rates. Although, in theory, an income tax cut, in form of a wage subsidy, could offset the effects of the shock, this would deteriorate fiscal finances leading to an increase in the government debt. Since in the Ricardian agents economy steady state level is already high, the policymaker’s optimal response is to raise taxes as a more effective way to mitigate the inflationary consequences of the shock. Given that

the fiscal side is mainly concerned with debt stabilization, by running fiscal surpluses, monetary policy is tightened to help ensure that inflation rate returns to its steady state.

**Limited asset markets participation ( $\lambda = 0.3$ ).** The presence of limited asset markets participation amplifies the inflationary consequences of a markup shock. As can be seen in the IRFs, when the fraction of liquidity constrained agents in the economy is set to 30%, the initial surge in inflation is higher than it was on the Ricardian agents case, reflecting the fact that when distortions are deepened, the discretionary policymaker is more willing to accept a higher rate of inflation to temper the fall in output. The most striking result is that monetary policy's optimal response to the shock is to set the nominal interest rate in a manner which leads to an almost pegged real rate of return. This, in turn, leaves the burden of stabilization of the economy against the consequences of the disturbance almost entirely to fiscal policy. The reason for this is that while variations in the real rate of return affect solely the decisions of Ricardian households, fiscal instruments have a direct impact on both agents in the model economy and, thus, constitute a more effective way in mitigating the inflationary consequences caused by the rise in markup. Moreover, numerical results show that government expenditures are hardly used as an instrument of either macroeconomic or fiscal stabilization<sup>18</sup>. This is due to the facts that: (i) government consumption enters directly in the utility functions of both agents; (ii) under the benchmark calibration, the elasticity of substitution for government consumption is low; and (iii) public expenditures crowds out private consumption (Galí et al., 2007). Therefore, the key instrument used to stabilize both inflation and government debt is the distortionary tax rate.

The immediate response of the optimal policy mix following the shock is to lower distortionary taxes. By cutting taxes, the policymaker offsets the inflationary consequences of the shock, since this lowers real marginal costs of production. As a direct consequence of tax reductions, government debt levels initially rise. Nevertheless, time consistency constraint requires that debt has to return to its steady state level and, thus, after some time, there is an overshooting in labor taxes to ensure that the upward trend in public debt is reversed and starts to converge to its long run equilibrium value.

To summarize, the coexistence of limited asset markets participation and a low steady state level of debt-to-GDP, in the case of  $\lambda = 0.3$ , gives fiscal policy a much more active role in macroeconomic stabilization. Under an almost fixed real rate of return, the burden of both offsetting the inflationary consequences of a markup shock and debt stabilization rests entirely on distortionary taxation.

**Limited asset markets participation ( $\lambda = 0.5$ ).** When the fraction of liquidity constrained consumers is increased even further to half of the model economy's population, the optimal responses of the policy mix to a markup shock are changed. In this particular case, the steady state level of public debt is negative (as shown in Subsection 6.1). With the government accumulating a stock of assets, rather than liabilities, the policymaker now faces a tradeoff: while inflation surprises boosts the output moving it closer to the efficient level, it also deteriorates the real value of those assets. The discretionary policymaker, then, has to balance those opposing forces when deciding how to move its instruments in response to the shock. The optimal IRFs to a markup shock, thus, imply decreases on the rate of real return and the net stock of nominal assets held by the government and a surge in distortionary tax rates. Fiscal instruments are optimally chosen to reduce the stock of assets in order to compensate the losses on their real value impugned by the hike in the inflation rate. Note that although this could be obtained by tax cuts, as in the previous case, now half of the consumers population spends all of their disposable income and, therefore, tax cuts would exacerbate the inflationary pressures, demanding an even higher deaccumulation of assets. Monetary policy accommodates

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<sup>18</sup>Government spending movements are largely in line with variations in output, yielding a stable ratio of government consumption-to-GDP.

this more active role of fiscal policy by lowering the rate of real return, displaying the inverse aggregate demand logic of [Bilbiie \(2008\)](#). Nevertheless, as we shall see, this passive-like behavior of monetary policy is dependent on the steady state level of government debt.

**Remark 1.** *Numerical results show that the optimal time-consistent policy mix's choice of instruments on macroeconomic and fiscal stabilization to the shock is largely dependent on the share of rule-of-thumb consumers in the model economy. For values of  $\lambda$  greater than 0.3, the conventional macroeconomic prescription is reversed and monetary policy's behavior resembles a passive policy while the activism of fiscal policy is enhanced. Nonetheless, those results are also dependent on the level of government debt. Figure 4 shows the non-linearities implied by the policy decisions as functions of lagged debt. The optimal decision rules suggest that for higher levels of public debt the conventional policy assignments are restored. In other words, once debt breaches some threshold, monetary policy responds to the shock by assuring that the real rate of return on debt rises and, thus, the conventional slope of aggregate demand is obtained.*

**Remark 2.** *We conjecture that those results rely on the assumption of a joint discretionary policy. In this case, a single policymaker has full control of all policy instruments in the economy. However, nowadays most economies delegate monetary policy to an instrument independent central bank and fiscal policy to the finance ministry or the legislature. It would be interesting to expand the present analysis to the case where there are interactions between independent authorities. We leave that for future research.*

### 6.3 The role of maturity structure of debt

The effects of the maturity structure of debt on the optimal policy mix under the presence of limited asset market participation are considered in Table 3. Setting all the parameters at their values on the baseline parameterization, we fix the fraction of liquidity constrained consumers at  $\lambda = 0.3$  and allow the coupon decay parameter to assume values in the following set  $\rho \in \{0; 0.7588; 0.9598; 0.9849\}$  corresponding, respectively, to 1 quarter, 1 year, 5 years and 10 years debt maturity.

The first column of the table considers the conventional assumption found in the literature that debt's duration is only single period. Given the model parameterization, when  $\rho = 0$  debt is of quarterly maturity. Under this assumption, the steady state debt-to-GDP ratio is almost zero and the annualized inflation rate reaches 4.14%.

As previously discussed in Section 4, a longer maturity of debt reduces the debt stabilization bias and in doing so allows the policymaker to sustain a higher debt-to-GDP ratio in the steady state. This can be seen at Table 3 as we gradually rise debt maturity to one, five and ten years. The results show that the steady state debt-to-GDP ratio is an increasing function of the debt maturity structure, reflecting the inverse relationship between longer-term debt and the debt stabilization bias. As the level of debt drives the endogenous inflationary bias problem, the steady state rate of inflation follows the upward trend, going rising from 4.14% (in annual terms) under quarterly maturity to 5.35% when debt maturity is 10 years.

In a similar model, [Leeper et al. \(2019\)](#) show that when debt maturity rises from one quarter to ten years, debt-to-GDP ratio exhibits an accentuated increase from -11.1% to 53.6%. The presence of limited asset market participation in the economy makes the steady state debt stock less sensitive to variations in maturity. Our numerical results show that  $\frac{bP^M}{4Y}$  rises from 0.49% to 19.8%, over the same range. The reason for this flatter upward trend when the economy deviates from the Ricardian agents scenario is the following: notwithstanding the fact that an increase in the maturity of debt reduces the debt stabilization bias, when the share of liquidity constrained consumers in the economy rises, this leads to a more pronounced

Variable	1 qtr maturity $\rho = 0$	1 yr maturity $\rho = 0.7588$	5 yr maturity $\rho = 0.9598$	10 yr maturity $\rho = 0.9849$
$\frac{bPM}{4Y}$	0.49%	6.44%	12.6%	19.8%
$(\Pi^4 - 1)$	4.14%	4.47%	4.88%	5.35%
$(R^4 - 1)$	6.25%	6.58%	7.01%	7.48%
$Y$	1.032	1.031	1.031	1.031
$G/Y$	19.4%	19.3%	19.4%	19.4%
$\tau$	20.4%	20.4%	20.6%	20.8%
$C^o$	0.844	0.845	0.844	0.844
$C^r$	0.802	0.801	0.800	0.798
$C$	0.831	0.832	0.831	0.830
$N^o$	1.021	1.020	1.020	1.020
$N^r$	1.057	1.057	1.058	1.058
$\frac{\Xi}{1-\lambda}$	0.070	0.069	0.069	0.069
$V^o$	-346.3	-346.2	-346.2	-346.1
$V^r$	-366.9	-367.3	-367.7	-368.2
$V$	-352.5	-352.5	-352.6	-352.7

Table 3: Steady state: the role of debt maturity structure ( $\lambda = 0.3$ ).

monopolistic distortion, pushing the bias in the opposite direction. Therefore, in a LAMP model the reduction in the debt stabilization bias is lower in magnitude, yielding a flatter upward trend in the steady state debt-to-GDP ratio as the maturity increases<sup>19</sup>.

#### 6.4 Welfare analysis

In this subsection we consider the welfare effects that variations on the fraction of liquidity constrained consumers and on the degree of myopia of the policymaker impose on the model economy. A closer inspection of Table 2 shows that different values of the share of limited asset market participation,  $\lambda$ , have distinct impacts on the value functions of optimizing,  $V^o$ , and rule-of-thumb agents,  $V^r$ , suggesting distributional consequences. Furthermore, as [Mankiw \(2000\)](#) pointed out, a higher level of debt increases steady state inequality between agents. We address this issue by varying the degree of myopic behavior of the policymaker,  $\delta$ , given that this affects the steady state level of government debt.

We follow [Adam and Billi \(2008\)](#); [Albonico and Rossi \(2017\)](#) in adopting a measure for the utility losses associated to different parameterization scenarios. The percentage loss in terms of consumption of alternative scenarios is calculated with respect to the deterministic steady state of some given benchmark parameterization. Let  $V^B$  denotes the utility for the benchmark's deterministic steady state, given by:

$$V^B = \frac{1}{(1-\beta)} [\lambda u(C^r, G, N^r) + (1-\lambda)u(C^o, G, N^o)], \quad (23)$$

where  $u(C^j, G, N^j)$  for  $j \in \{o, r\}$  is the period utility defined in equation (1). If  $V^A$  is the value function of an alternative scenario evaluated at the deterministic steady state,  $u(C_A^j, G_A, N_A^j)$ ,

<sup>19</sup>When we set  $\lambda$  to zero and, thus, consider the case in which all households are Ricardians, debt-to-GDP ratio rises from 4.71% when debt maturity is of single period to 73.7% under 10 years debt maturity. If half of the consumers' population is liquidity constrained,  $\lambda = 0.5$ , numerical results show an increase in the net stock of nominal assets (rather than liabilities) from 1.09% to 9.2%.

the permanent reduction in private consumption,  $\mu^A$ , that would imply the benchmark deterministic steady state to be welfare equivalent to the alternative scenario is implicitly defined by the following expression:

$$V^A = \frac{1}{1-\beta}[\lambda u(C^r(1+\mu^A), G, N^r) + (1-\lambda)u(C^o(1+\mu^A), G, N^o)]. \quad (24)$$

The same formulae are used to evaluate the value functions of each type of consumer, i.e.,  $V_j^B = u(C^j, G, N^j)/(1-\beta)$ , and:

$$V_j^A = \frac{1}{1-\beta}[u(C^j(1+\mu^A), G, N^j)].$$

Firstly we look at the welfare effects that variations in the fraction of liquidity constrained consumers have under the baseline calibration detailed in Table 1, assuming  $\lambda = 0.3$  as a benchmark scenario. The welfare losses (or gains) for each type of consumer relative to their benchmark values, in terms of consumption equivalents, under different specifications for  $\lambda$  are reported in Table 4.

	RAE ( $\lambda = 0$ )	$\lambda = 0.5$
Ricardians, $V^o$	-2.69	3.04
Liquidity Constrained, $V^r$	-	0.59
Total, $V$	-0.03	-0.03

Table 4: Welfare losses in consumption equivalents (percentages).

Numerical results suggest that while total welfare is only marginally affected by variations in  $\lambda$ , reflecting the fact that the policymaker aims to maintain the overall welfare of the economy, increases in the share of rule-of-thumb agents represent welfare gains for both types of consumers. Optimizing agents experience a welfare a welfare gain when  $\lambda$  rises due to the fact that their financial wealth is enhanced allowing a boost in consumption. In comparison with the benchmark scenario, the Ricardian welfare gain is of 3.04% when  $\lambda$  rises to half of the population, and there is a welfare loss of 2.69% when  $\lambda = 0$  and the model collapses to the standard Ricardian agents economy (RAE). Welfare of rule-of-thumb agents also raises when  $\lambda$  increases. The reason for that is, since a higher fraction of liquidity constrained consumers reduces the steady state level of debt, this allows a smaller income tax rate compared to the benchmark scenario and, thus, consumption of rule-of-thumb agents rise. Note, however, that this is the only source of gains for liquidity constrained consumers, while Ricardian agents can support an even higher level of consumption since their financial wealth also increases. Therefore, a higher degree of limited asset market participation brings welfare gains for both Ricardian and rule-of-thumb agents but, at the same time, increases the steady state inequality.

Turning now to the distributional effects that the level of government debt has on the economy, Table 5 shows the welfare losses in percentage terms of consumption equivalents of a high government debt level scenario in relation to the baseline calibration of Section 5. Reported results distinguishes between total, Ricardian and liquidity constrained welfare and between different values of  $\lambda$ .

A more myopic behavior of the policymaker - lowering the value of  $\delta$  - serves to render the equilibrium steady state of debt-to-GDP ratio higher. Under the alternative scenario considered in this exercise,  $\delta$  is set to 0.9808 which implies a time horizon for the policymaker of approximately 13 years. This raises the steady state value of debt to 113% in the Ricardian agents

	RAE ( $\lambda = 0$ )	$\lambda = 0.3$	$\lambda = 0.5$
Total	-0.44	-0.33	-0.27
Ricardians	-0.44	0.04	0.29
Liquidity Constrained	-	-1.14	-0.78

Table 5: Level of debt and welfare losses in consumption equivalents (percentages)

economy, 55.9% when  $\lambda = 0.3$  and 20.7% in the case of  $\lambda = 0.5$ . The logic runs as follows: reductions in the policy maker’s time horizon yield the debt stabilization bias smaller once he is less inclined to incur the costs of debt reduction in order to achieve long-term benefits (Leeper et al., 2019).

The numerical results reported in Table 5 show that when there is a switch from the benchmark case to the high government debt level scenario, total welfare shrinks for all values of  $\lambda$ . In the Ricardian agents economy,  $\lambda = 0$ , this reduction is caused by the fall in consumption of optimizing agents, as a higher level of debt means a higher level of taxation to help the financing of this debt. Nevertheless, when the share of rule-of-thumb consumers in the economy rises, there is an associated increase in the financial wealth of Ricardian agents as now per capita profits are higher. The rise in the financial wealth more than offsets the losses caused by higher tax rates and, thus, the welfare of Ricardian consumers rises in both cases when  $\lambda = 0.3$  and  $\lambda = 0.5$ . Since liquidity constrained households do not have access to asset markets, they experience a pronounced fall in consumption induced by the higher taxation prevailing under higher levels of debt. Therefore, welfare of rule-of-thumb agents decreases as the level of debt rises.

To summarize, the level of government debt influences the distribution of income and consumption in our model economy in the presence of limited asset market participation. Higher levels of debt mean a higher level of taxation. The distortionary labor income tax rate falls both on optimizing and liquidity constrained consumers, but only Ricardian agents receive interest payments on government bonds. Therefore, when the level of debt increases there is a redistribution effect raising the steady state inequalities in the model economy. This corroborates the results found by Mankiw (2000) in his savers-spenders model.

## 7 Conclusion

The recent global financial crisis and its developments were associated with a steadily increase in the public debt-to-GDP ratios and sharp reductions on the availability of credit to households in advanced economies. To cope with this scenario, the present work studied jointly optimal and time-consistent monetary and fiscal policies in the presence of limited asset markets participation. A standard New Keynesian model is augmented to incorporate the government’s budget constraint, where government spending is financed by distortionary taxation and/or issuing of long term debt, and the existence of a share of consumers who do not smooth consumption intertemporally. A single policymaker has full control of all policy instruments and is constrained to act in a time-consistent manner. The inability to commit yields the inflationary bias problem state-dependent and also creates an incentive to bring back government debt to its steady state, following a shock. In order to deal with the state-dependencies caused by discretionary policy, the model is solved using global solution and nonlinear techniques.

We found that the presence of limited asset markets participation, by increasing the distortions in the model economy, impacts the long-run equilibrium values of relevant macroeconomic

variables. More specifically, increases in the share of liquidity constrained agents are associated with lower steady state rate of inflation and public debt-to-GDP ratio. Moreover, when a fraction of consumers cannot smooth consumption intertemporally, the upward trend in the steady state debt-to-output ratio caused by the issuing of longer-term government debt is weakened. The optimal responses of stabilization policy in face of a shock to the firms' markup are largely dependent on both the size of government debt and the fraction of rule-of-thumb consumers. Finally, the share of liquidity constrained agents and the size of debt both cause redistribution effects amongst agents and, therefore, influence steady state inequalities.

The current work can be extended in several ways, which include:

1. To take capital accumulation explicitly into consideration. As [Galí et al. \(2004\)](#) point out, in disregarding capital accumulation, the only difference in behavior across Ricardian and liquidity constrained agents is the fact that optimizing households earn dividend incomes from the ownership of firms. In this case, ultimately, all shares of profits have to be held by Ricardian consumers in equal proportions.
2. To consider redistributive fiscal policies. In the current work we disregard the possibility for fiscal authority to rely on different fiscal transfers across households. [Albonico and Rossi \(2015\)](#) show that fully redistributive fiscal policy eliminates the extra inflationary bias caused by the presence of limited asset markets participation, but at the cost of reducing Ricardian agents' welfare. Redistributive policies can also be useful in tempering the rise of steady state inequalities caused by higher levels of government debt.
3. To consider stabilization issues when macroeconomic policy is delegated to independent and potentially strategic fiscal and monetary authorities.

## A Figures

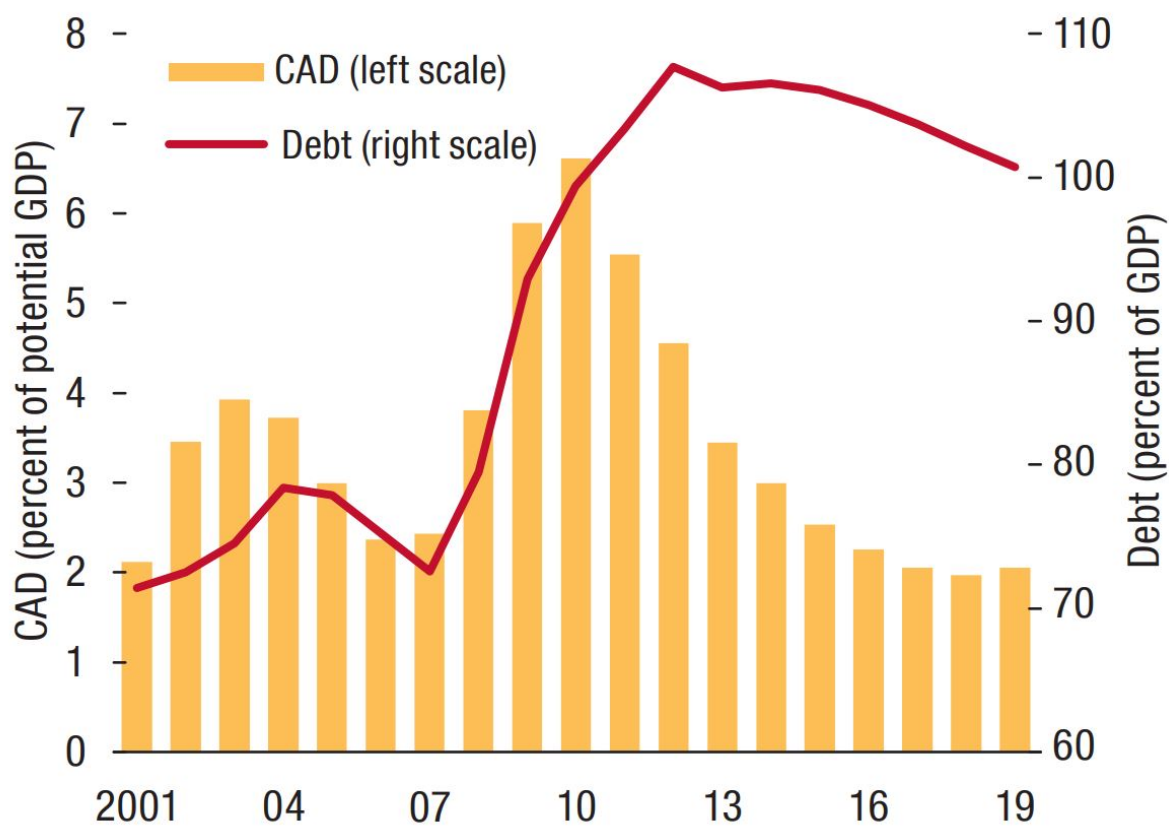
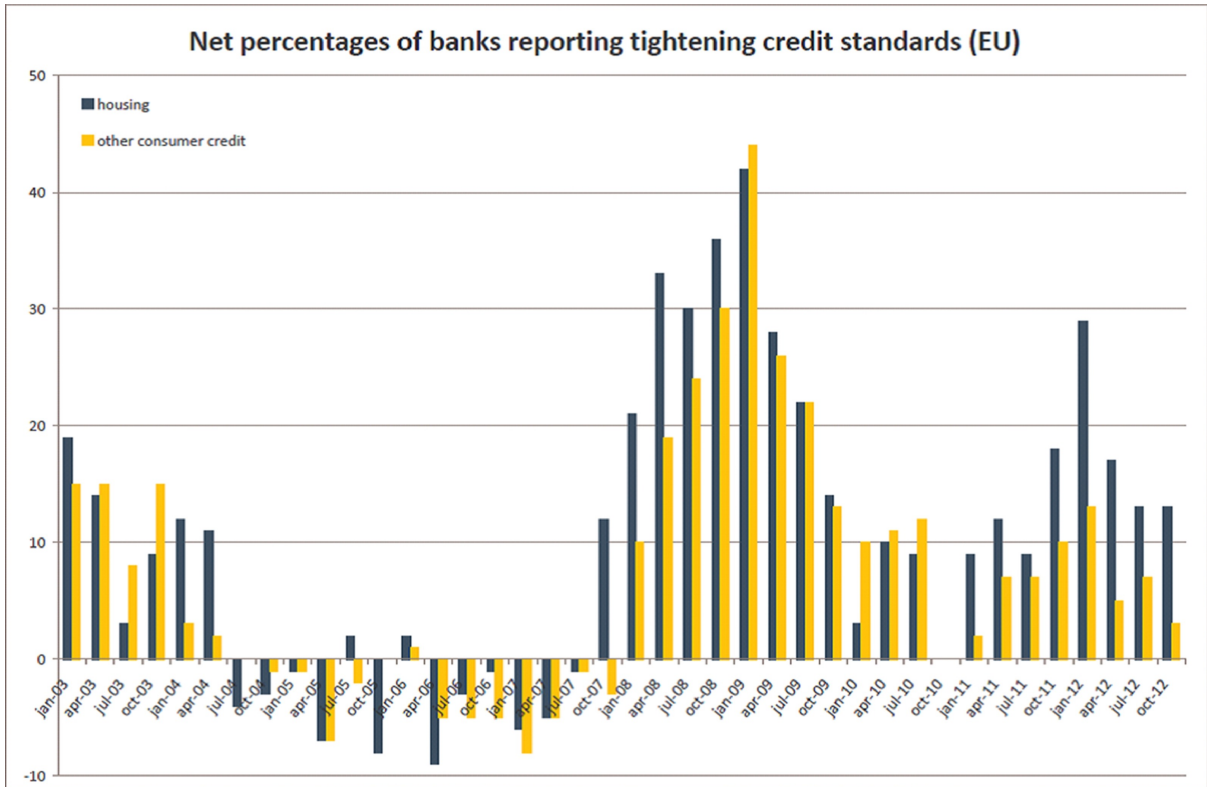
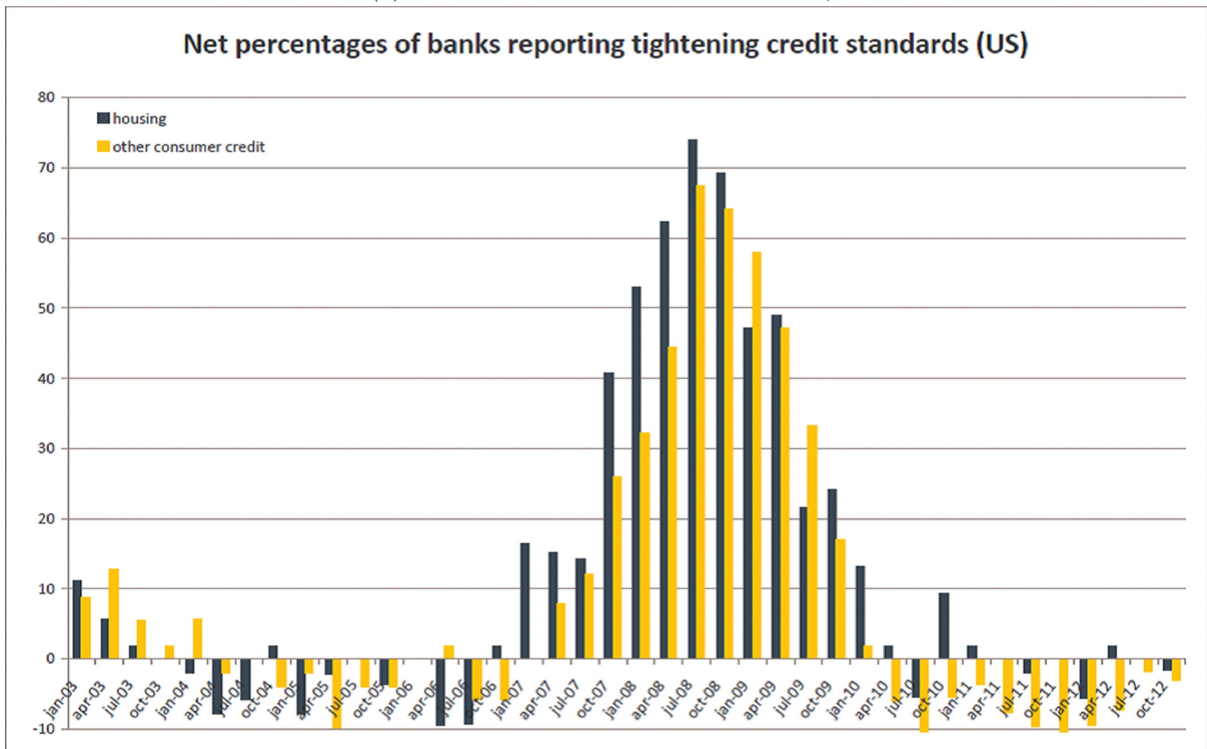


Figure 2: Debt-to-GDP ratio and cyclically adjusted deficit (CAD) as percentage of potential GDP in advanced economies. Source: [International Monetary Fund \(2014\)](#).





(a) Credit standards in the EU economy.



(b) Credit standards in the US economy

Figure 3: Net percentages of banks reporting tightening credit standards in the EU and US economies. Source: [Albonico and Rossi \(2017\)](#).

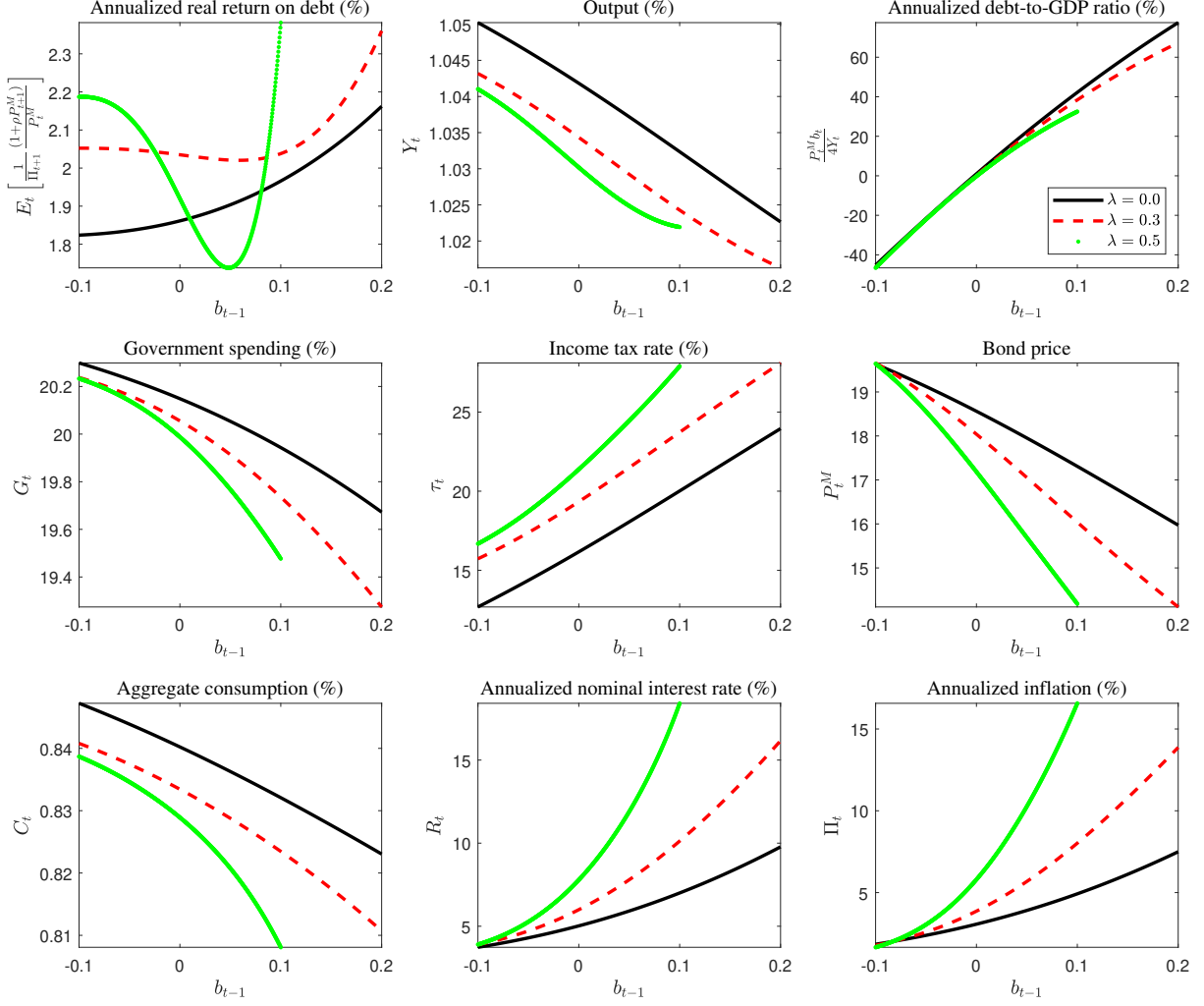


Figure 4: Policy rules as functions of lagged debt under the benchmark calibration. The grid for the elasticity of substitution between varieties is fixed at  $\bar{\epsilon}$ .

## B Numerical Algorithm

In this appendix we outline the Chebyshev collocation method with time iteration employed to solve the model described in the paper. This method can be framed under a more general approach, namely, projection methods. Projection methods handle DSGE models by building a basis function, indexed by fixed coefficients, that approximates a policy function in order find the vector of coefficients which minimizes some given residual function. For textbook treatments of the numerical methods, see [Miranda and Fackler \(2004\)](#); [Judd \(1998\)](#); [Fernández-Villaverde et al. \(2016\)](#). Our exposition heavily relies on the description of the techniques found in [Leeper et al. \(2019\)](#).

Define the vector of states  $s_t = (b_{t-1}, \epsilon_t)$ , where  $b_{t-1}$  is the real stock of debt, which is endogenous, and  $\epsilon_t$  is the elasticity of substitution between varieties, assumed to be an exogenously given process. The law of motion for each state variable is given by:

$$\begin{aligned} P_t^M b_t &= (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \tau_t w_t N_t + G_t, \\ \ln(\epsilon_t) &= (1 - \rho_\epsilon) \ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \epsilon_t, \quad \epsilon_t \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1), \end{aligned}$$

where  $0 \leq \rho_\epsilon < 1$ .

We approximate the following 16 functional equations associated with 10 endogenous variables and 6 Lagrange multipliers,  $\{C_t^o(s_t), C_t^r(s_t), N_t^o(s_t), N_t^r(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \mu_{1t}(s_t), \mu_{2t}(s_t), \mu_{3t}(s_t), \mu_{4t}(s_t), \mu_{5t}(s_t), \mu_{6t}(s_t)\}$ . In order to collect the policy functions of endogenous variables, we define a function  $X : \mathbb{R}^2 \rightarrow \mathbb{R}^{16}$ , where:

$$X(s_t) = (C_t^o(s_t), C_t^r(s_t), N_t^o(s_t), N_t^r(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \mu_{1t}(s_t), \mu_{2t}(s_t), \mu_{3t}(s_t), \mu_{4t}(s_t), \mu_{5t}(s_t), \mu_{6t}(s_t)).$$

In doing so, the equilibrium conditions of the model can be rewritten compactly as:

$$\Gamma(s_t, X(s_t), \mathbb{E}_t[Z(X(s_{t+1}))], \mathbb{E}_t[Z_b(X(s_{t+1}))]) = 0,$$

where  $\Gamma : \mathbb{R}^{2+16+3+3} \rightarrow \mathbb{R}^{16}$ , and:

$$Z(X(s_{t+1})) = \begin{bmatrix} Z_1(X(s_{t+1})) \\ Z_2(X(s_{t+1})) \\ Z_3(X(s_{t+1})) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \epsilon_{t+1}) \\ L(b_t, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}(1 + \rho P_{t+1}^M)\mu_{3t+1} \end{bmatrix},$$

with:

$$\begin{aligned} M(b_t, \epsilon_{t+1}) &= (C_{t+1}^o)^{-\sigma} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*} \left( \frac{\Pi_{t+1}}{\Pi^*} - 1 \right), \\ L(b_t, \epsilon_{t+1}) &= (C_{t+1}^o)^{-\sigma} (\Pi_{t+1})^{-1} (1 + \rho P_{t+1}^M), \end{aligned}$$

and:

$$Z_b(X(s_{t+1})) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial L(b_t, \epsilon_{t+1})}{\partial b_t} \\ \frac{\partial [(\Pi_{t+1})^{-1}(1 + \rho P_{t+1}^M)\mu_{3t+1}]}{\partial b_t} \end{bmatrix}.$$

Taking the derivatives yields:

$$\begin{aligned} M_b(b_t, \epsilon_{t+1}) &\equiv \frac{\partial M(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma (C_{t+1}^o)^{-\sigma-1} Y_{t+1} \frac{\Pi_{t+1}}{\Pi^*} \left( \frac{\Pi_{t+1}}{\Pi^*} - 1 \right) \frac{\partial C_{t+1}^o}{\partial b_t} \\ &+ (C_{t+1}^o)^{-\sigma} \frac{\Pi_{t+1}}{\Pi^*} \left( \frac{\Pi_{t+1}}{\Pi^*} - 1 \right) \frac{\partial Y_{t+1}}{\partial b_t} + (C_{t+1}^o)^{-\sigma} \frac{Y_{t+1}}{\Pi^*} \left( \frac{2\Pi_{t+1}}{\Pi^*} - 1 \right) \frac{\partial \Pi_{t+1}}{\partial b_t}, \end{aligned}$$

and

$$\begin{aligned} L_b(b_t, \epsilon_{t+1}) &\equiv \frac{\partial L(b_t, \epsilon_{t+1})}{\partial b_t} = -\sigma (C_{t+1}^o)^{-\sigma-1} (\Pi_{t+1})^{-1} (1 + \rho P_{t+1}^M) \frac{\partial C_{t+1}^o}{\partial b_t} \\ &- (C_{t+1}^o)^{-\sigma} (\Pi_{t+1})^{-2} (1 + \rho P_{t+1}^M) \frac{\partial \Pi_{t+1}}{\partial b_t} + \rho (C_{t+1}^o)^{-\sigma} (\Pi_{t+1})^{-1} \frac{\partial P_{t+1}^M}{\partial b_t}. \end{aligned}$$

As in [Leeper et al. \(2019\)](#), we are relying on the Interchange of Integration and Differentiation Theorem and assuming that  $\mathbb{E}_t[Z_b(X(s_{t+1}))] = \partial \mathbb{E}_t[Z(X(s_{t+1}))] / \partial b_t$ . To solve the model, we then use projection methods to find a vector-valued function  $X$  that  $\Gamma$  maps to some ‘‘approximately’’ zero function.

In order to easy notation, we follow the convention in the literature by using  $s(b, \epsilon)$  to denote the current state of the economy, and  $s'$  to represent the next period state.

The Chebyshev collocation algorithm to solve the nonlinear system describing the model economy can be described as follows:

1. Define the collocation nodes and the space of linearly independent basis functions to approximate the policy functions:

- Choose an order of approximation<sup>20</sup> (i.e., the polynomial degrees)  $n_b$  and  $n_\epsilon$  for each

<sup>20</sup>In this work we have used polynomials of 11<sup>th</sup> degree for each state variable.

dimension of the state space  $s = (b, \epsilon)$ , then there are  $N_s = (n_b + 1) \times (n_\epsilon + 1)$  nodes. Let  $S = (S_1, S_2, \dots, S_{N_s})$  denote the set of collocation nodes.

- Compute the  $n_b + 1$  and  $n_\epsilon + 1$  zeros of the Chebyshev polynomials of order  $n_b + 1$  and  $n_\epsilon + 1$  as:

$$z_b^i = \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \quad i = 1, 2, \dots, n_b + 1.$$

$$z_\epsilon^i = \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \quad i = 1, 2, \dots, n_\epsilon + 1.$$

Chebyshev polynomials have the convenient property that they are smooth and bounded between  $[-1, 1]$ . Besides, their roots are quadratically clustered toward  $\pm 1$ .

- Given that the domain of Chebyshev polynomials is  $[-1, 1]$  and that the state variables of our DSGE model are different, we should use some form of linear translation. In order to do that, we compute the collocation points  $\epsilon_i$  as:

$$\epsilon_i = \frac{\epsilon_{\max} + \epsilon_{\min}}{2} + \frac{\epsilon_{\max} - \epsilon_{\min}}{2} z_\epsilon^i = \frac{\epsilon_{\max} - \epsilon_{\min}}{2} (z_\epsilon^i + 1) + \epsilon_{\min}, \quad i = 1, 2, \dots, n_\epsilon + 1,$$

which map  $[-1, 1]$  onto  $[\epsilon_{\min}, \epsilon_{\max}]$ . Similarly, the collocation points  $b_i$  are:

$$b_i = \frac{b_{\max} + b_{\min}}{2} + \frac{b_{\max} - b_{\min}}{2} z_b^i = \frac{b_{\max} - b_{\min}}{2} (z_b^i + 1) + b_{\min}, \quad i = 1, 2, \dots, n_b + 1,$$

mapping  $[-1, 1]$  onto  $[b_{\min}, b_{\max}]$ . Note that:

$$S = \{(b_i, \epsilon_j) | i = 1, 2, \dots, n_b + 1, j = 1, 2, \dots, n_\epsilon + 1\},$$

are the tensor grids, with  $S_1 = (b_1, \epsilon_1), S_2 = (b_1, \epsilon_2), \dots, S_{N_s} = (b_{n_b+1}, \epsilon_{n_\epsilon+1})$ .

- The space of the approximating functions, denoted as  $\Omega$ , is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_1))) & T_0(\xi(b_1)T_2(\xi(\epsilon_1))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_1))) \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_2))) & T_0(\xi(b_1)T_2(\xi(\epsilon_2))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_2))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_1)T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_1(\xi(\epsilon_{n_\epsilon+1}))) & T_0(\xi(b_{n_b+1})T_2(\xi(\epsilon_{n_\epsilon+1}))) & \cdots & T_{n_b}(\xi(b_{n_b+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))) \end{bmatrix}_{N_s \times N_s}$$

where  $\xi(x) = 2\frac{x-x_{\min}}{x_{\max}-x_{\min}} - 1$  is a linear translation mapping the domain of  $x \in [x_{\min}, x_{\max}]$  onto  $[-1, 1]$ , and  $T$  defines the Chebyshev polynomials recursively, with  $T_0(x) = 1, T_1(x) = x$ , and the general  $n + 1$ -th order polynomial is given by:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

- Then, at each node  $s \in S$ , the policy functions  $X(s)$  are approximated by  $X(s) = \Omega(s)\Theta_X$ , where

$$\Theta_X = [\theta^{c^o}, \theta^{c^r}, \theta^{n^o}, \theta^{n^r}, \theta^y, \theta^\Pi, \theta^b, \theta^\tau, \theta^{\bar{p}}, \theta^G, \theta^{\mu_1}, \theta^{\mu_2}, \theta^{\mu_3}, \theta^{\mu_4}, \theta^{\mu_5}, \theta^{\mu_6}],$$

is a  $N_S \times 16$  matrix of coefficients.

2. Formulate an initial guess for the matrix of coefficients,  $\Theta_X^0$ , and specify some convergence criterion  $\epsilon_{\text{tol}}$ . We set  $\epsilon_{\text{tol}} = 10^{-8}$ .
3. At each iteration  $j$ , the matrix of coefficients is updated  $\Theta_X^j$  by implementing the following time iteration procedure:

- For each collocation node  $s \in S$ , compute the possible values of future policy functions  $X(s')$  for  $k = 1, \dots, q$ . That is:

$$X(s') = \Omega(s')\Theta_X^{j-1}.$$

where  $q$  is the number of nodes in a Gauss-Hermite quadrature<sup>21</sup>. Note that:

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\epsilon}(xi(\epsilon')),$$

is a  $q \times N_s$  matrix, for  $j_b = 0, \dots, n_b$  and  $j_\epsilon = 0, \dots, n_\epsilon$ , with  $b' = \hat{b}(s; \theta^b)$  and:

$$\ln(\epsilon') = (1 - \rho_\epsilon) \ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon) + z_k \sqrt{2\sigma_\epsilon^2}.$$

The two auxiliary functions can be calculated in a similar way:

$$\begin{aligned} M(s') &\approx \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \hat{Y}(s'; \theta^y) \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} \left(\frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} - 1\right), \\ L(s') &\approx \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \left(\hat{\Pi}(s'; \theta^\Pi)\right)^{-1} \left(1 + \frac{\rho P^{\hat{M}}(s'; \theta^{\bar{p}})}{\Pi^* - \rho\beta}\right). \end{aligned}$$

Following [Leeper et al. \(2016, 2019\)](#), in the numerical analysis we approximate the function  $\tilde{P}_t^M = (\Pi^* - \rho\beta)P_t^M$  rather than  $P_t^M$ , since the former is less sensitive to variations in the maturity structure. The hat notation indicates that the policy functions are only approximated.

- Let  $\omega_k$  denote the weights of the Gauss-Hermite quadrature, the expectation terms can be calculated, at each node  $s$ , as:

$$\begin{aligned} \mathbb{E}[M(s')] &\approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^q \omega_k \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \hat{Y}(s'; \theta^y) \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} \left(\frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} - 1\right) \equiv \bar{M}(s', q), \\ \mathbb{E}[L(s')] &\approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^q \omega_k \left(\hat{C}^o(s'; \theta^{c^o})\right)^{-\sigma} \left(\hat{\Pi}(s'; \theta^\Pi)\right)^{-1} \left(1 + \frac{\rho P^{\hat{M}}(s'; \theta^{\bar{p}})}{\Pi^* - \rho\beta}\right) \equiv \bar{L}(s', q), \\ \mathbb{E}_t \left[ \left( \frac{1 + \rho P_{t+1}^M}{\Pi_{t+1}} \right) \mu_{3t+1} \right] &\approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^q \omega_k \left( \frac{1 + \frac{\rho P^{\hat{M}}(s'; \theta^{\bar{p}})}{\Pi^* - \rho\beta}}{\hat{\Pi}(s'; \theta^\Pi)} \right) \hat{\mu}_3(s'; \theta^{\mu_3}) \equiv \Lambda(s', q). \end{aligned}$$

Hence:

$$\mathbb{E}[Z(X(s'))] \approx \mathbb{E}[\hat{Z}(X(s'))] = \begin{bmatrix} \bar{M}(s', q) \\ \bar{L}(s', q) \\ \Lambda(s', q) \end{bmatrix}.$$

<sup>21</sup>In this work we set an 12-point Gauss-Hermite quadrature rule.

- To calculate the partial derivatives under expectations,  $\mathbb{E}[Z_b(X(s'))]$ , we obtain the following terms:

$$\begin{aligned}\frac{\partial C_{t+1}^o}{\partial b_t} &\approx \sum_{j_b=0}^{n_b} \sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta_{j_b j_\epsilon}^{c^o}}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b')) T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{C}_b^o(s'), \\ \frac{\partial Y_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b} \sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta_{j_b j_\epsilon}^y}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b')) T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{Y}_b(s'), \\ \frac{\partial \Pi_{t+1}}{\partial b_t} &\approx \sum_{j_b=0}^{n_b} \sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta_{j_b j_\epsilon}^\Pi}{b_{\max} - b_{\min}} T'_{j_b}(\xi(b')) T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{\Pi}_b(s'), \\ \frac{\partial P_{t+1}^M}{\partial b_t} &\approx \sum_{j_b=0}^{n_b} \sum_{j_\epsilon=0}^{n_\epsilon} \frac{2\theta_{j_b j_\epsilon}^{\bar{p}}}{(b_{\max} - b_{\min})(\Pi^* - \rho\beta)} T'_{j_b}(\xi(b')) T_{j_\epsilon}(\xi(\epsilon')) \equiv \hat{P}_b^M(s').\end{aligned}$$

Hence, the partial derivatives under expectations can be approximated as:

$$\begin{aligned}& \frac{\partial \mathbb{E}[M(s')]}{\partial b} \\ & \approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^q \omega_k \left[ \begin{aligned} & -\sigma \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma-1} \hat{Y}(s'; \theta^y) \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} \left( \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} - 1 \right) \hat{C}_b^o(s') \\ & + \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma} \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} \left( \frac{\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} - 1 \right) \hat{Y}_b(s') \\ & + \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma} \frac{\hat{Y}(s'; \theta^y)}{\Pi^*} \left( \frac{2\hat{\Pi}(s'; \theta^\Pi)}{\Pi^*} - 1 \right) \hat{\Pi}_b(s') \end{aligned} \right] \\ & \equiv \hat{M}_b(s', q), \\ & \frac{\partial \mathbb{E}[L(s')]}{\partial b} \\ & \approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^q \omega_k \left[ \begin{aligned} & -\sigma \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma-1} \left( \hat{\Pi}(s'; \theta^\Pi) \right)^{-1} \left( 1 + \frac{\rho \hat{P}^M(s'; \theta^{\bar{p}})}{\Pi^* - \rho\beta} \right) \hat{C}_b^o(s') \\ & - \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma} \left( \hat{\Pi}(s'; \theta^\Pi) \right)^{-2} \left( 1 + \frac{\rho \hat{P}^M(s'; \theta^{\bar{p}})}{\Pi^* - \rho\beta} \right) \hat{\Pi}_b(s') \\ & + \rho \left( \hat{C}^o(s'; \theta^{c^o}) \right)^{-\sigma} \left( \hat{\Pi}(s'; \theta^\Pi) \right)^{-1} \hat{P}_b^M(s') \end{aligned} \right] \\ & \equiv \hat{L}_b(s', q).\end{aligned}$$

That is,

$$\mathbb{E}[Z_b(X(s'))] \approx \mathbb{E}[\hat{Z}_b(X(s'))] = \begin{bmatrix} \hat{M}_b(s', q) \\ \hat{L}_b(s', q) \end{bmatrix}.$$

4. At each collocation node  $s$ , solve the following functional for  $X(s)$ :

$$\Gamma(s, X(s), \mathbb{E}[\hat{Z}(X(s'))], \mathbb{E}[\hat{Z}_b(X(s'))]) = 0,$$

using some routine to solve systems of nonlinear equations. Once  $X(s)$  is obtained, the matrix of coefficients can be calculated as follows:

$$\hat{\Theta}_X^j = (\Omega(S)^T \Omega(S))^{-1} \Omega(S)^T X(S).$$

5. Update the approximating coefficients,  $\Theta_X^j = \eta \hat{\Theta}_X^j + (1 - \eta) \Theta_X^{j-1}$ , where  $0 \leq \eta \leq 1$  is a dampening parameter used for improving convergence.
6. If  $\|\Theta_X^j - \Theta_X^{j-1}\| < \epsilon_{\text{tol}}$ , the algorithm converged. Otherwise, restart the procedure from Step 3.

## C Euler equation errors

To assess the accuracy of our numerical solutions, we follow [Judd \(1992\)](#) in performing Euler equation errors (EEE) analyses to address the difference between the exact and the approximated solutions. As it is convention in the literature, we calculate the  $\log_{10} |EEE(b_{t-1}, \epsilon_t)|$ , for a discussion see [Fernández-Villaverde et al. \(2016\)](#). Table 6 summarizes some basic statistics about the Euler equation errors for the numerical results presented in Subsection 6.1, the most common reported statistics for the EEE are the mean of the Euler equation errors, in our case a simple average<sup>22</sup>, and the maximum of the EEE. The reported statistics were calculated using an evenly-spaced grid consisting of 40 points for the stock of debt,  $b_t$ , and 40 points for the elasticity of substitution between intermediate goods,  $\ln(\epsilon_t)$ . The results are similar on a finer grid<sup>23</sup>.

Table 6: Euler equation errors (EEE).

	Benchmark calibration			$\phi = 50$			$\frac{\epsilon-1}{\epsilon} = 6\%$		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$
$\log_{10} \max  EEE $	-6.41	-6.65	-6.50	-6.56	-6.53	-5.61	-6.16	-6.36	-2.26
$\log_{10} \text{mean}  EEE $	-7.78	-7.81	-7.79	-7.84	-7.74	-7.51	-7.70	-7.75	-5.64
$\log_{10} \text{median}  EEE $	-11.06	-10.28	-11.3	-11.6	-9.30	-8.73	-9.67	-9.47	-7.93
$\text{std}(EEE)$	5.20e-08	4.86e-08	-7.27	-7.38	-7.28	-7.02	-7.22	-7.27	-4.27

A visual representation of the accuracy of the approximated solutions is presented in Figure 5. In this figure, the model is solved under the benchmark calibration for  $\lambda = 0.3$ .

<sup>22</sup>Alternatively, some researchers use some estimate of the ergodic distribution of state variables.

<sup>23</sup>For all the other models considered, with different maturities and degrees of myopia, the accuracy of the approximations is similar.

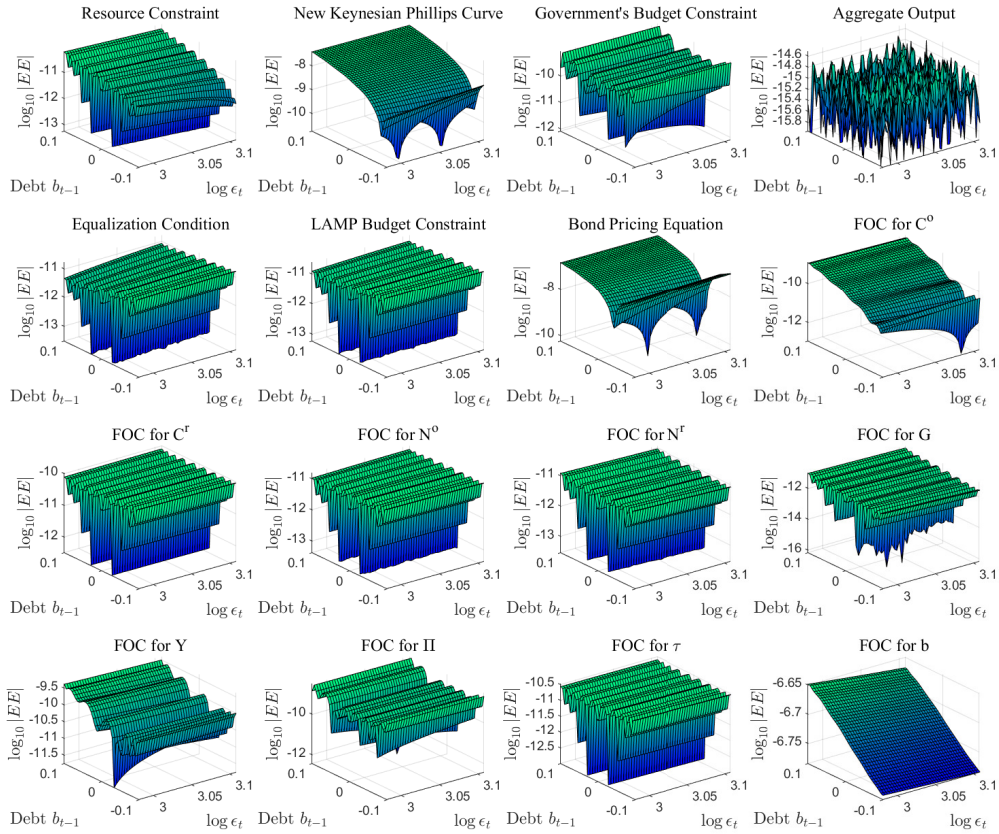


Figure 5: Euler Equation Errors (EEE): benchmark calibration with  $\lambda = 0.3$ .

## References

- Adam, K. and Billi, R. M. (2008). Monetary conservatism and fiscal policy. *Journal of Monetary Economics*, 55(8):1376–1388.
- Aiyagari, S. R., Marcet, A., Sargent, T. J., and Seppala, J. (2002). Optimal Taxation without State-Contingent Debt. *Journal of Political Economy*, 110(6):1220–1254.
- Albonico, A., Paccagnini, A., and Tirelli, P. (2014). Estimating a dsge model with limited asset market participation for the euro area. Working Papers 286, University of Milano-Bicocca, Department of Economics.
- Albonico, A., Paccagnini, A., and Tirelli, P. (2016). In search of the euro area fiscal stance. *Journal of Empirical Finance*, 39(PB):254–264.
- Albonico, A. and Rossi, L. (2015). Policy Games, Distributional Conflicts, And The Optimal Inflation. *Macroeconomic Dynamics*, 19(6):1261–1293.
- Albonico, A. and Rossi, L. (2017). Inflation bias and markup shocks in a lamp model with strategic interaction of monetary and fiscal policy. *Journal of Macroeconomics*, 52(C):39–55.



- Alesina, A. and Tabellini, G. (1990). A positive theory of fiscal deficits and government debt. *Review of Economic Studies*, 57(3):403–414.
- Alvarez, F., Lucas, R. E., and Weber, W. E. (2001). Interest Rates and Inflation. *American Economic Review*, 91(2):219–225.
- Ascari, G., Colciago, A., and Rossi, L. (2017). Limited Asset Market Participation, Sticky Wages, And Monetary Policy. *Economic Inquiry*, 55(2):878–897.
- Barro, R. (1979). On the determination of the public debt. *Journal of Political Economy*, 87(5):940–71.
- Beetsma, R. M. W. J. and Jensen, H. (2004). Mark-up fluctuations and fiscal policy stabilization in a monetary union. *Journal of Macroeconomics*, 26(2):357–376.
- Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.
- Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383 – 398.
- Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual 1989, Volume 4*, NBER Chapters, pages 185–246. National Bureau of Economic Research, Inc.
- Chadha, J. S. and Nolan, C. (2007). Optimal simple rules for the conduct of monetary and fiscal policy. *Journal of Macroeconomics*, 29(4):665 – 689.
- Debortoli, D. and Nunes, R. (2013). Lack Of Commitment And The Level Of Debt. *Journal of the European Economic Association*, 11(5):1053–1078.
- Di Bartolomeo, G. and Rossi, L. (2007). Effectiveness of monetary policy and limited asset market participation: Neoclassical versus keynesian effects. *International Journal of Economic Theory*, 3(3):213–218.
- Di Bartolomeo, G., Rossi, L., and Tancioni, M. (2011). Monetary policy, rule-of-thumb consumers and external habits: a g7 comparison. *Applied Economics*, 43(21):2721–2738.
- Eusepi, S. and Preston, B. (2013). Fiscal foundations of inflation: imperfect knowledge. Staff Reports 649, Federal Reserve Bank of New York.
- Fernández-Villaverde, J., Rubio-Ramírez, J. F., and Schorfheide, F. (2016). Solution and Estimation Methods for DSGE Models. volume 2 of *Handbook of Macroeconomics*, pages 527 – 724. Elsevier.
- Furlanetto, F. (2011). Fiscal stimulus and the role of wage rigidity. *Journal of Economic Dynamics and Control*, 35(4):512–527.
- Galí, J., López-Salido, J. D., and Vallés, J. (2004). Rule-of-thumb consumers and the design of interest rate rules. *Journal of Money, Credit and Banking*, 36(4):739–763.
- Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, 5(1):227–270.

- International Monetary Fund (2014). Fiscal monitor - back to work: How fiscal policy can help. Technical report, International Monetary Fund, Washington.
- Judd, K. L. (1992). Projection methods for solving aggregate growth models. *Journal of Economic Theory*, 58(2):410–452.
- Judd, K. L. (1998). *Numerical Methods in Economics*, volume 1 of *MIT Press Books*. The MIT Press.
- Judd, K. L. (2004). Existence, uniqueness, and computational theory for time consistent equilibria: A hyperbolic discounting example.
- Kirsanova, T., Satchi, M., Vines, D., and Wren-Lewis, S. (2007). Optimal fiscal policy rules in a monetary union. *Journal of Money, Credit and Banking*, 39(7):1759–1784.
- Klein, P., Krusell, P., and Ríos-Rull, J.-V. (2008). Time-Consistent Public Policy. *Review of Economic Studies*, 75(3):789–808.
- Klein, P. and Ríos-Rull, J.-V. (2003). Time-consistent optimal fiscal policy. *International Economic Review*, 44(4):1217–1245.
- Krusell, P., Kuruscu, B., and Smith, A. (2002). Equilibrium welfare and government policy with quasi-geometric discounting. *Journal of Economic Theory*, 105(1):42–72.
- Krusell, P., Martin, F. M., and Rios-Rull, J.-V. (2006). Time Consistent Debt. 2006 Meeting Papers 210, Society for Economic Dynamics.
- Leeper, E. M., Leith, C. B., and Liu, D. (2016). Optimal time-consistent monetary, fiscal and debt maturity policy. Working Papers 2016/04, Business School - Economics, University of Glasgow.
- Leeper, E. M., Leith, C. B., and Liu, D. (2019). Optimal time-consistent monetary, fiscal and debt maturity policy. NBER Working Papers 25658, National Bureau of Economic Research, Inc.
- Leith, C. and Liu, D. (2016). The inflation bias under Calvo and Rotemberg pricing. *Journal of Economic Dynamics and Control*, 73(C):283–297.
- Leith, C. and von Thadden, L. (2008). Monetary and fiscal policy interactions in a new keynesian model with capital accumulation and non-ricardian consumers. *Journal of Economic Theory*, 140(1):279–313.
- Leith, C. and Wren-Lewis, S. (2013). Fiscal Sustainability in a New Keynesian Model. *Journal of Money, Credit and Banking*, 45(8):1477–1516.
- Lucas, R. J. and Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, 12(1):55–93.
- Mankiw, N. G. (2000). The Savers-Spenders Theory of Fiscal Policy. *American Economic Review*, 90(2):120–125.
- Miranda, M. J. and Fackler, P. L. (2004). *Applied Computational Economics and Finance*, volume 1 of *MIT Press Books*. The MIT Press.

- Muscattelli, V. A., Tirelli, P., and Trecroci, C. (2004). Fiscal and monetary policy interactions: Empirical evidence and optimal policy using a structural New-Keynesian model. *Journal of Macroeconomics*, 26(2):257–280.
- Nisticò, S. (2016). Optimal Monetary Policy And Financial Stability In A Non-Ricardian Economy. *Journal of the European Economic Association*, 14(5):1225–1252.
- Ortigueira, S. (2006). Markov-Perfect Optimal Taxation. *Review of Economic Dynamics*, 9(1):153–178.
- Persson, T. and Svensson, L. (1989). Why a stubborn conservative would run a deficit: Policy with time-inconsistent preferences. *The Quarterly Journal of Economics*, 104(2):325–345.
- Preston, B. and Eusepi, S. (2011). The maturity structure of debt, monetary policy and expectations stabilization. 2011 Meeting Papers 1287, Society for Economic Dynamics.
- Rigon, M. and Zanetti, F. (2018). Optimal Monetary Policy and Fiscal Policy Interaction in a Non-Ricardian Economy. *International Journal of Central Banking*, 14(3):389–436.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.
- Sargent, T. J. and Wallace, N. (1981). Some unpleasant monetarist arithmetic. *Quarterly Review*, (Fall).
- Sims, E. and Wolff, J. (2017). State-dependent fiscal multipliers: Calvo vs. Rotemberg. *Economics Letters*, 159(C):190–194.
- Siu, H. (2004). Optimal fiscal and monetary policy with sticky prices. *Journal of Monetary Economics*, 51(3):575–607.
- Stehn, S. J. and Vines, D. (2008). Strategic Interactions between an Independent Central Bank and a Myopic Government with Government Debt. IMF Working Papers 08/164, International Monetary Fund.
- Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking*, 33(3):669–728.
- Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32(2):137–150.